Video 3.2A Transcript - Sum and Difference of Two Angles

This video presentation is to demonstrate how to use the sum and difference formula for sine and cosine. If we want to find an exact value such as cosine of fifteen degree, we will need to use the sum or the difference of two angles for sine or cosine. Since fifteen degrees is not listed on the unit circle, so we will have to figure out a way to use the unit circle for us to find the exact value of cosine of fifteen degrees. So, think of it in this way, where we know if I have a cosine of fifteen degrees which we can reprint as cosine of sixty degree minus forty-five degree. To use the unit circle, we need to think about the angles on the unit circle that can somehow help us to come up with the fifteen degrees. So, we can think of the sixty degrees minus the forty-five degrees. So, to use the difference of two angles for cosine, the formula says it gonna be cosine of alpha (which is our sixty degree) times the cosine of beta which is my second angle (forty-five degree) plus the sine of the sixty degree times the sine of forty-five degree. If you can recall, on the unit circle cosine of sixty degree is half… times forty-five degree. Forty-five degree on the unit circle will be square root of two over two… plus the sine of sixty degree is square root of three over two times sine of forty-five degree is also square root of two over two. So, if we multiply half times square root of two over two, we should get square root of two over four… plus square root of three over two times square root of two over two… should be square root of six over four. So, since the denominators are the same the least common denominator will be four. So, if we will add the numerator, that will just be square root of two plus square root of six over the least common denominator is four. So, the exact value of cosine of fifteen degree will be square root of two plus square root of six over four.

Now let's do an example with <u>sine</u> of fifteen degrees. We can use the same technique, just use a different formula. To find the exact value of sine of fifteen degree we can say sine of fifteen degree is same as sine of sixty degree minus forty-five degree. To use the formula, sine of first angle minus the second angle, is sine of the first angle times cosine of the second angle. Sine of my sixty degrees is the first angle, times cosine of my second angle forty-five degree, the formula says minus the cosine of sixty degree (the first angle) times the sine of forty-five degrees. Let me scroll back up… sine of first angle times cosine of second angle minus cosine the first angle times sine of the second angle (called beta here). So, sine of sixty degree on the unit circle will be square root of three over two times cosine of forty-five degree: square root of two over two minus cosine of sixty degree (which is a half) times sine of forty-five degree (square root of two over two). Now, if we multiply these, that will be square root of six over four minus square root of two over four. Since the least common denominator is already the same (which is four), we can just subtract the numerators square root of six minus square root of two over four. So, to find the exact value of the sine of fifteen degree, we will use the angles on the unit circle that will somehow come out with the fifteen degree. So our trick here is using sixty degree minus forty-five degree which is equivalent to fifteen degree, and use the difference of two angles for sine. Sine of fifteen degree exact value will be square root of six minus square root of two over four, and that will conclude this video presentation.