4

Oblique Triangles

Objectives

Students will be able to:

- Define the three key terms regarding oblique triangles.
- Apply the Law of Sines to find the missing two sides of an oblique triangle when given one side and two angles.
- Determine if a side-side-angle (SSA) case produces one triangle or no triangle at all.
- Apply the Law of Cosines to find the missing side and the two missing angles of an oblique triangle when given two sides and one angle.
- Apply the Law of Cosines to find the three missing angles of an oblique triangle.
- Approximate the area of a triangle by applying Heron's Formula.

Orienting Questions

✓ What are the definitions of the three key terms in this module?
✓ How are the missing two sides of an oblique triangle found when given one side and two angles?
✓ How is one triangle or no triangle at all determined?
✓ How is the Law of Cosines used to find the missing side and the two missing angles of an oblique triangle?
✓ How are the missing three angles of an oblique triangle found using the Law of Cosines?
✓ What is Heron's Formula?

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INTRODUCTION

Recall from Module 1 that the word "trigonometry" means "measurement of triangles". Thus far, we have only been focusing on the study of right triangles. In this module, we will examine the relationships of the sides and angles of oblique triangles.

4.1 THE LAW OF SINES

An ***oblique triangle*** is any triangle whose angles do not contain a right angle. Examples of oblique triangles are acute triangles or obtuse triangles (See figure 4.1a and 4.2b).

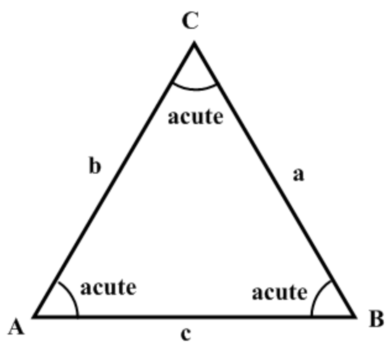


Figure 4.1a: An oblique triangle where three angles are acute angles.

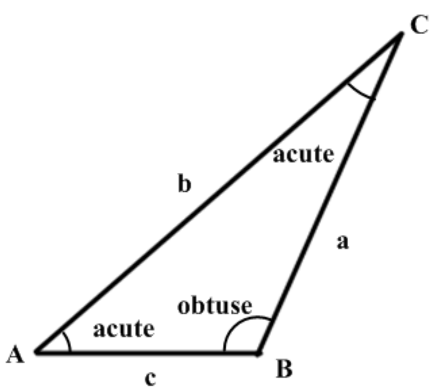


Figure 4.1b: An oblique triangle where one angle is an obtuse angle and the other two angles are acute angles.

In figure 4.1a and 4.1b, the angles are labeled as A,B,C and the corresponding sides, that are opposite to these angles, are labeled as a,b,c. One of the important relationships between the angles and sides of an oblique triangle is called the Law of Sines. The ***Law of Sines*** is the ratio of the lengths of the side to the sine of the angle opposite to that side. The ratio is the same for all three sides of a triangle (See table 4.1a).

Table 4.1a

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

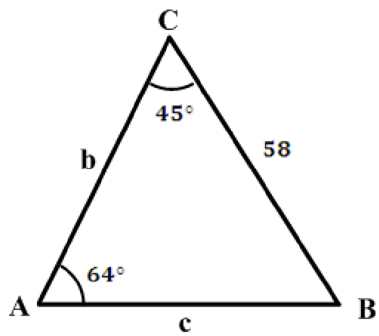
The Law of Sines is used to solve an oblique triangle when two angles and one side is known. By applying the Law of Sines, we can find the missing sides and angle.

There are two cases where two angles and one side is known. The two cases we will examine are side-angle-angle (SAA) and angle-side-angle (ASA).

EXAMPLES

Please work through the following examples before completing the 4.1 LEARNING ACTIVITY:

Example 1: Find the missing sides and the missing angle in the case of SAA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.



Since the sum of interior angles of a triangle is $180°$, we can subtract $45°$ and $64°$ from $180°$ to find $\angle B$.

$$\angle B = 180° - 45° - 64°$$

$$\angle B = 71°$$

To find side b, we apply the Law of Sines by using the given $\angle A$ and side a.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$\frac{58}{\sin 64°} = \frac{b}{\sin 71°}$ apply cross product

$b \cdot \sin 64° = 58 \cdot \sin 71°$ divide both sides by $\sin 64°$

$b = \frac{58 \cdot \sin 71°}{\sin 64°}$ set the calculator in degree mode

$b = 61.0$

To find side c, we apply the Law of Sines by using the given $\angle A$ and side a.

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{58}{\sin 64°} = \frac{c}{\sin 45°}$ apply cross product

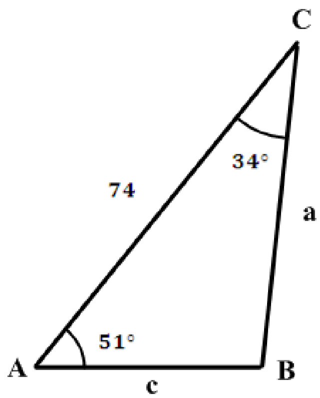
$c \cdot \sin 64° = 58 \cdot \sin 45°$ divide both sides by $\sin 64°$

$c = \frac{58 \cdot \sin 45°}{\sin 64°}$ set the calculator in degree mode

$c = 45.6$

So, $\angle B = 71°$, $b = 61.0$, and $c = 45.6$.

Example 2: Find the missing sides and the missing angle in the case of ASA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.



To find $\angle B$, we can subtract $51°$ and $34°$ from $180°$.

$\angle B = 180° - 51° - 34°$

$\angle B = 95°$

To find side a, we apply the Law of Sines by using the given side b and $\angle B$ that we just found.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 51°} = \frac{74}{\sin 95°} \quad \text{apply cross product}$$

$$a \cdot \sin 95° = 74 \cdot \sin 51° \quad \text{divide both sides by } \sin 95°$$

$$a = \frac{74 \cdot \sin 51°}{\sin 95°} \quad \text{set the calculator in degree mode}$$

$$a = 57.7$$

To find side c, we apply the Law of Sines by using the given side b and $\angle B$ that we just found.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{74}{\sin 95°} = \frac{c}{\sin 34°} \quad \text{apply cross product}$$

$$c \cdot \sin 95° = 74 \cdot \sin 34° \quad \text{divide both sides by } \sin 95°$$

$$c = \frac{74 \cdot \sin 34°}{\sin 95°} \quad \text{set the calculator in degree mode}$$

$$c = 41.5$$

So, $\angle B = 95°$, $a = 57.7$, and $c = 41.5$.

4.1 LEARNING ACTIVITY

a. Find the missing sides and the missing angle in the case of SAA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$A = 31°, C = 130°, a = 14$

b. Find the missing sides and the missing angle in the case of ASA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$B = 54°, C = 82°, a = 13$

4.1.1 SIDE-SIDE-ANGLE (SSA) CASE

Another case where we can apply the Law of Sines is the side-side-angle (SSA). In the SSA case, the sides and the angle given can result in the formation of one triangle, one right triangle, no triangle, or two triangles. The number of triangles that can be formed, if any, will depend on the length of the height of the triangle where $h = b \cdot \sin A$ (See figure 4.1.1a through 4.1.1 d).

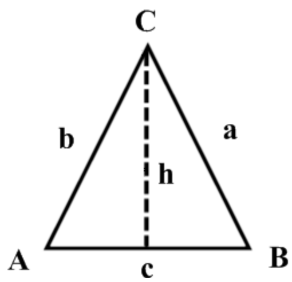


Figure 4.1.1a: One triangle where $a > h$ and $a > b$.

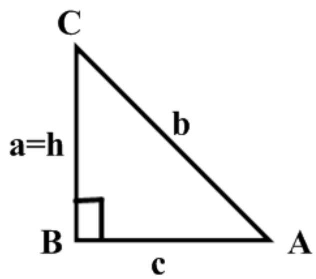


Figure 4.1.1b: One right triangle where $a = h$.

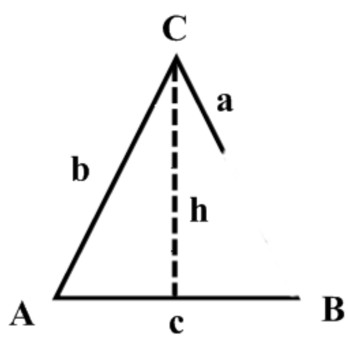


Figure 4.1.1c: No triangle formed where $a < h$ and $a < b$.

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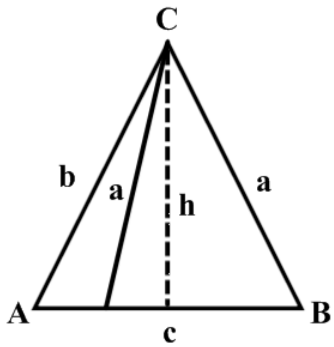


Figure 4.1.1d: Two triangle formed where $a > h$ and $a < b$.

If side a is greater than height h and side a is greater than side b, then one triangle is produced. If side a is equal to the height, then a right triangle is produced. If side a is less than height h, then a triangle can not be produced. If side a is greater than height h and side a is less than side b, then there are two triangles produced.

EXAMPLES

Please work through the following examples before completing the 4.1.1 LEARNING ACTIVITY:

Example 1: Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.

$\boldsymbol{a = 20, b = 15, A = 41°}$

To determine what triangle will be produced, use $h = b \cdot \sin A$.

$h = 15 \cdot \sin 41°$ set the calculator in degree mode

$h = 9.8$

Since side a is greater than height h and side a is greater than side b, a triangle will be produced. Now, we will find the missing side and angles.

To find $\angle B$, we apply the Law of Sines by using the given side a and $\angle A$.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{20}{\sin 41°} = \frac{15}{\sin B}$$ apply cross product

$$20 \cdot \sin B = 15 \cdot \sin 41°$$ divide both sides by 20

$$\sin B = \frac{15 \cdot \sin 41°}{20}$$

$\sin B = 0.4920$ to find the $\angle B$, use sine inverse on the calculator

$\angle B = \sin^{-1}(0.4920)$ set the calculator in degree mode

$\angle B = 29°$

Since sine of angle B is equal to a positive ratio, angle B can also be in the second quadrant. So, $\angle B = 180° - 29° = 151°$ in the second quadrant.

To find $\angle C$, we can subtract $41°$ and $29°$ from $180°$ or subtract $41°$ and $151°$ from $180°$.

$\angle C = 180° - 41° - 29°$ or $\angle C = 180° - 41° - 150°$

$\angle C = 110°$ *or* $\angle C = -11°$

Since angle C can not be negative, angle B can not be $150°$.

To find side c, we apply the Law of Sines by using the given side a, $\angle A$, and $\angle C$ that we just found.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 41°} = \frac{c}{\sin 110°}$$ apply cross product

$c \cdot \sin 41° = 20 \cdot \sin 110°$ divide both sides by $\sin 41°$

$$c = \frac{20 \cdot \sin 110°}{\sin 41°}$$ set the calculator in degree mode

$c = 28.6$

So, $\angle B = 29°$, $\angle C = 110°$ and $c = 28.6$.

Example 2: Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.

$a = 10, b = 30, A = 130°$

To determine what triangle will be produced, use $h = b \cdot \sin A$.

$h = 10 \cdot \sin 130°$ set the calculator in degree mode

$h = 22$

Since side a is less than height h and side a is less than side b, no triangle is produced.

4.1.1 LEARNING ACTIVITY

a. Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 10, b = 8, A = 40°$

b. Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 5, b = 20, A = 45°$

4.2 THE LAW OF COSINES

Another important relationship between the sides and the angles of an oblique triangle is the Law of Cosines (See table 4.2a). The ***Law of Cosines*** is the square of a side of the triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle. The Law of Cosines is used for the cases where two sides and an included angle (SAS) are known and where three side (SSS) are known.

Table 4.2a

The Law of Cosines

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$

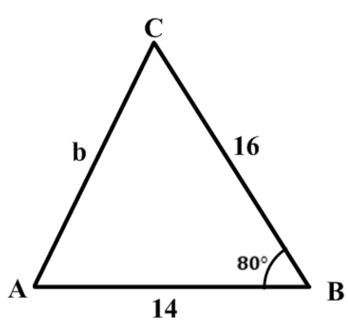
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos B$

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos C$

EXAMPLES

Please work through the following examples before completing the 4.2 LEARNING ACTIVITY:

Example 1: Find the missing side and angles in the SAS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.



To find the missing side, use the Law of Cosines $b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos B$.

$b^2 = 16^2 + 14^2 - 2 \cdot 16 \cdot 14 \cdot \cos 80°$ set the calculator in degree mode

$b^2 = 374.2056164$ take the square root on both sides

$b = 19.3$

To find $\angle A$, we apply the Law of Sines by using the given side a, $\angle B$ and side b we just found.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{16}{\sin A} = \frac{19.3}{\sin 80°} \quad \text{apply cross product}$$

$$19.3 \cdot \sin A = 16 \cdot \sin 80° \quad \text{divide both sides by } 19.3$$

$$\sin A = \frac{16 \cdot \sin 80°}{19.3} \quad \text{set the calculator in degree mode}$$

$$\sin A = .8164$$

$$\angle A = \sin^{-1}(0.8164)$$

$$\angle A = 55°$$

To find $\angle C$, we can subtract $80°$ and $55°$ from $180°$.

$$\angle C = 180° - 80° - 55°$$

$$\angle C = 45°$$

Example 2: Find the missing side and angles in the SSS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.

$a = 8, b = 8, c = 6$

To find $\angle A$, we apply the Law of Cosines.

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$$

$$8^2 = 8^2 + 6^2 - 2 \cdot 8 \cdot 6 \cdot \cos A$$

$$64 = 64 + 36 - 96 \cdot \cos A$$

$$64 = 100 - 96 \cdot \cos A \quad \text{subtract 64 and add } -96 \cdot \cos A \text{ on both sides}$$

$$96 \cdot \cos A = 36 \quad \text{divide both sides by 96}$$

$$\cos A = \frac{36}{96}$$

$$\angle A = \cos^{-1}\left(\frac{36}{96}\right)$$

$$\angle A = 68°$$

To find $\angle B$, we apply the Law of Sines by using the given side a, b and $\angle A$ we just found.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{8}{\sin 68°} = \frac{8}{\sin B} \quad \text{apply cross product}$$

$$8 \cdot \sin B = 8 \cdot \sin 68° \quad \text{divide both sides by 8}$$

$$\sin B = \frac{8 \cdot \sin 68°}{8} \quad \text{set the calculator in degree mode}$$

$$\sin B = 0.9272$$

$$\angle B = \sin^{-1}(0.9272)$$

$$\angle B = 68°$$

To find $\angle C$, we can subtract both $68°$ from $180°$.

$$\angle C = 180° - 68° - 68°$$

$$\angle C = 44°$$

So, $\angle A = 68°$, $\angle B = 68°$ and $\angle C = 44°$.

4.2 LEARNING ACTIVITY

a. Find the missing side and angles in the SAS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$c = 30, B = 41°, a = 26.5$

b. Find the missing side and angles in the SSS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 9, b = 6, c = 4$

4.3 AREA OF AN OBLIQUE TRIANGLE

Recall that the basic area formula for a triangle is $A = \frac{1}{2} \cdot b \cdot h$. The formula requires a side and a height of a triangle. What if the height is not given directly? Instead, if we are given two sides and an angle or three sides of a triangle, we can use the area of an oblique triangle formula or Heron's Formula to the find the area (See table 4.3a).

Table 4.3a

Area of An Oblique Triangle

$$Area = \frac{1}{2} \cdot b \cdot c \cdot \sin A$$

$$Area = \frac{1}{2} \cdot a \cdot b \cdot \sin C$$

$$Area = \frac{1}{2} \cdot a \cdot c \cdot \sin B$$

Heron's Formula

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Where s is one-half its perimeter: $s = \frac{1}{2}(a + b + c)$.

VIDEO 4.3A

Click here to watch examples of how to find the area of an oblique triangle.

VIDEO 4.3A TRANSCRIPT

A video transcript on how to find the area of an oblique triangle is available under Module 4 in Moodle.

EXAMPLES

Please work through the following example before completing the 4.3 LEARNING ACTIVITY:

Example 1: Find the area of the triangle. Round the area to the nearest whole number.

$\boldsymbol{b = 10\ in, c = 24\ in, A = 62°}$

$$\boldsymbol{Area = \frac{1}{2} \cdot b \cdot c \cdot \sin A = \frac{1}{2} \cdot 10 \cdot 24 \cdot \sin 62°}$$

$$\boldsymbol{Area = 106\ in^2}$$

Example 2: Find the area of the triangle. Round the area to the nearest whole number.

$\boldsymbol{a = 8, b = 8, c = 6}$

To find the area, we will find one-half its perimeter first then use the Heron's Formula.

$$\boldsymbol{s = \frac{1}{2}(a + b + c)}$$

$$\boldsymbol{s = \frac{1}{2}(8 + 8 + 6) = 11}$$

$$\boldsymbol{Area = \sqrt{11(11-8)(11-8)(11-6)} = 22}$$

4.3 LEARNING ACTIVITY

a. Find the area of the triangle. Round the area to the nearest whole number.
   $a = 20\ ft, b = 16\ ft, C = 109°$

b. Find the area of the triangle. Round the area to the nearest whole number.
   $a = 9m, b = 6m, c = 4m$

MAJOR CONCEPTS

KEY CONCEPTS

Use the Law of Sines for SSA and SAS cases.

Use the Law of Cosines for SAS and SSS cases.

SSA case can produce one triangle, one right triangle, no triangle, or two triangles.

The Area of An Oblique Triangle Formula, or Heron's Formula, can approximate the area of a triangle without the height given.

KEY TERMS

Oblique Triangle	Law of Sines	Law of Cosines

GLOSSARY

An ***oblique triangle*** is any triangle whose angles do not contain a right angle.

The ***Law of Sines*** is the ratio of the lengths of the side to the sine of the angle opposite to that side.

The ***Law of Cosines*** is the square of sides of the triangle equals the sum of the squares of the other two sides minus twice the product times the cosine of the included angle.

ASSESSMENT

ANSWERS TO LEARNING ACTIVITIES

4.1 LEARNING ACTIVITY

a. Find the missing sides and the missing angle in the case of SAA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$A = 31°, C = 130°, a = 14$
$\boldsymbol{Ans: \angle B = 19°, b = 8.8, c = 20.8}$

b. Find the missing sides and the missing angle in the case of ASA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$B = 54°, C = 82°, a = 13$
$\boldsymbol{Ans: \angle A = 44°, b = 15.1, c = 18.5}$

4.1.1 LEARNING ACTIVITY

a. Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 10, b = 8, A = 40°$
$\boldsymbol{Ans: One\ triangle. \angle B = 31°, \angle C = 109°, c = 14.7}$

b. Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 5, b = 20, A = 45°$
$\boldsymbol{Ans: No\ triangle\ produced}$

4.2 LEARNING ACTIVITY

a. Find the missing side and angles in the SAS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$c = 30, B = 41°, a = 26.5$
$\boldsymbol{Ans: b = 20.1, \angle A = 60°, \angle C = 79°}$

b. Find the missing side and angles in the SSS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 9, b = 6, c = 4$
$\boldsymbol{Ans: \angle A = 127°, \angle B = 32°, \angle C = 21°}$

4.3 LEARNING ACTIVITY

a. Find the area of the triangle. Round the area to the nearest whole number.
$a = 20\ ft, b = 16\ ft, C = 109°$
***Ans*: 151 ft^2**

b. Find the area of the triangle. Round the area to the nearest whole number.
$a = 9m, b = 6m, c = 4m$
***Ans*: 10 m^2**

MODULE REINFORCEMENT

SHORT ANSWER QUESTIONS

1) Find the missing sides and the missing angle in the case of ASA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$A = 40°, B = 100°, c = 20$

2) Find the missing sides and the missing angle in the case of ASA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$A = 85°, B = 76°, c = 1200$

3) Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 29, b = 19, A = 49°$

4) Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles. If a triangle is produced, find the missing side and angles. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 10, b = 40, A = 40°$

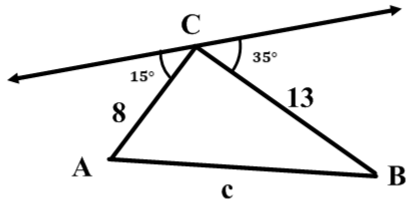
5) Find the missing side and angles in the SAS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$b = 4\ miles, A = 46°, c = 8\ miles$

6) Find the missing angles in the SSS case. Round the missing angles to the nearest degree.
$a = 4, b = 7, c = 6$

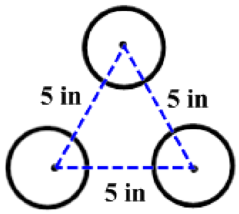
7) Find the area of the triangle. Round the area to the nearest whole number.
$a = 6\ in, b = 4\ in, C = 96°$.

8) Find the area of the triangle. Round the area to the nearest whole number.
$a = 13\ mi, b = 9\ mi, c = 5\ mi.$

9) Find the area of the figure below. Round to the nearest whole number.



10) Suppose three holes are drilled that are equally spaced between them to form a triangle. Find the area of the triangle. Round to the nearest whole number.



MULTIPLE CHOICE: READ THE FOLLOWING QUESTIONS OR STATEMENTS AND SELECT THE BEST ANSWER.

1) __________ Find the missing sides and the missing angle in the case of SAA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$A = 42°, C = 98°, c = 10$

a. $\angle B = 40°, a = 0.7, b = 0.7$
b. $\angle B = 40°, a = 6.8, b = 6.5$
c. $\angle B = 40°, a = 6.5, b = 6.8$

2) __________ Find the missing sides and the missing angle in the case of ASA. Round the missing sides to the nearest tenth and the missing angle to the nearest degree.
$A = 85°, B = 50°, c = 170$

a. $\angle C = 45°, a = 258.2, b = 198.5$
b. $\angle C = 45°, a = 120.7, b = 156.9$
c. $\angle C = 45°, a = 239.5, b = 184.2$

3) __________ Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles.
$a = 10, b = 8.9, A = 63°$

a. a triangle
b. a right triangle
c. no triangle
d. two triangles

4) __________ Determine if the two sides and an angle (SSA) produce a triangle, a right triangle, no triangle, or two triangles.
$a = 20, b = 45, A = 20°$

a. a triangle
b. a right triangle
c. no triangle
d. two triangles

5) __________ Find the missing side and angles in the SAS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$b = 7, A = 120°, c = 8$

a. $a = 13, B = 106°, C = 45°$
b. $a = 13, B = 28°, C = 32°$

c. $a = 13, B = 43°, C = 17°$

6) __________ Find the missing side and angles in the SSS case. Round the missing side to the nearest tenth and the missing angles to the nearest degree.
$a = 6.3, b = 2.2, c = 5$

a. $\angle A = 18°, \angle B = 49°, \angle C = 113°$
b. $\angle A = 18°, \angle B = 113°, \angle C = 49°$
c. $\angle A = 49°, \angle B = 18°, \angle C = 113°$
d. $\angle A = 113°, \angle B = 18°, \angle C = 49°$

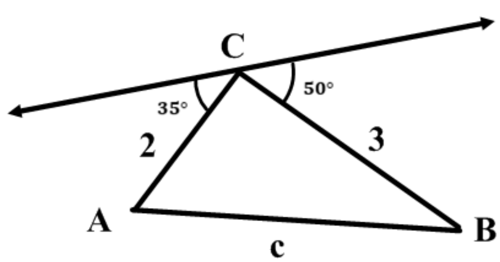
7) __________ Find the area of the triangle. Round the area to the nearest whole number.
$a = 6\ mm, c = 8\ mm, B = 32°$..

a. $13mm^2$
b. $20mm^2$
c. $15mm^2$

8) __________ Find the area of the triangle. Round the area to the nearest whole number.
$a = 11\ yd, b = 7\ yd, c = 9\ yd$

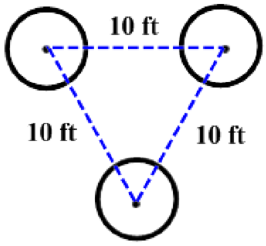
a. $29\ yd^2$
b. $30\ yd^2$
c. $31\ yd^2$
d. $32\ yd^2$

9) __________ Find the area of the figure below. Round to the nearest whole number.



a. $2\ square\ units$
b. $3\ square\ units$
c. $4\ square\ units$
d. $5\ square\ unts$

10) __________ Find the area of the figure below. Round to the nearest whole number.



a. $40\ ft^2$
b. $41\ ft^2$
c. $42\ ft^2$
d. $43\ ft^2$

ANSWER KEY

Short Answer Questions	Multiple Choice
1) $\angle C = 40°, b = 30.6, a = 20$ 2) $\angle C = 19°, a = 3671.8, b = 3576.4$ 3) $\angle B = 30°, \angle C = 101°, c = 37.7$ 4) $No\ triangle\ produced$ 5) $\mathrm{a} = 6.0, \angle B = 29°, \angle C = 105°$ 6) $\angle A = 35°, \angle B = 86°, \angle C = 59°$ 7) $12in^2$ 8) $16\ mi^2$ 9) $40\ square\ unit$ 10) $11\ in^2$	1) b 2) c 3) a 4) c 5) b 6) d 7) a 8) c 9) b 10) d

CRITICAL THINKING

1) If the side-side-angle given below produces two triangles, find all the missing sides and angles. Round the missing sides to the nearest tenth and the missing angles to the nearest degree.
$a = 20, b = 45, A = 20°$

$\boldsymbol{Ans}$:

$\boldsymbol{when\ \angle B = 50°, \angle C = 110°, c = 54.9}$

$\boldsymbol{when\ \angle B = 130°, \angle C = 30°, c = 29.2}$

2) What is the reason side-side-angle (SSA) is called the ambiguous case?

$\boldsymbol{Answers\ may\ vary.}$
$\boldsymbol{Side - side - angle\ is\ called\ the\ ambiguous\ case\ because\ the\ case\ may\ produce}$
$\boldsymbol{one\ triangle, one\ right\ triangle, no\ triangle, or\ two\ triangles.}$

3) Explain why the Law of Sines can not be used in the side-angle-side (SAS) case?
$\boldsymbol{Answers\ may\ vary..}$
$\boldsymbol{Regardless\ which\ two\ sides\ and\ the\ angle\ given, there\ is\ still\ a\ missing\ side}$
$\boldsymbol{and\ missing\ angle\ when\ seting\ the\ problem\ up\ in\ the\ proportion.}$

4) Argue the reasons why neither the Law of Sines nor the Law of Cosines can be used to find the sides of an oblique triangle when all three angles are known?
$\boldsymbol{Answers\ may\ vary.}$
$\boldsymbol{Both\ the\ Law\ of\ Sines\ and\ the\ Law\ of\ Cosines\ require\ side\ or\ sides\ in\ the\ formula.}$

So, without side or sides, we can not use both laws.

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