5

Polar Coordinates

Objectives

Students will be able to:

- Define the four key terms regarding polar coordinates.
- Plot polar coordinates on the polar coordinate system.
- Convert between rectangular and polar coordinates.
- Convert between polar and rectangular equations.
- Use the graphing calculator to graph circles, limacons, rose curves, and lemniscates.

Orienting Questions

- ✓ What are the definitions of the four key terms in this module?
- ✓ How are points in the polar coordinate system plotted?
- ✓ How are $x = r \cdot \cos\theta$ and $y = r \cdot \sin\theta$ used to convert between rectangular and polar coordinates?
- ✓ What equations are used to help convert between polar and rectangular equations?
- ✓ What are the steps of graphing polar equations?

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INTRODUCTION

Recall that the Unit Circle, with a radius of one unit, is plotted on the rectangular coordinate system. If we wish to plot a circle with a longer radius, we can easily plot it on a polar coordinate system. In this module, we will learn how to plot polar coordinates and polar equations in the polar coordinate system. We will also learn how trigonometric functions can be graphed on the polar coordinate system.

5.1 PLOT POLAR COORDINATES

A *polar coordinate* system is just like a Unit Circle on the rectangular coordinate system where the *x*-axis becomes a polar axis, and the origin becomes a pole (See figure 5.1a).

Figure 5.1a: A polar axis where the origin is called a pole.

A *polar axis* is a ray that extends to the right of the polar coordinate system. The *pole* is the endpoint of the polar axis. The polar coordinate (r, θ) is made up of a distance r from the pole to a point P and the angle θ is from the polar axis to the point P (See figure 5.1b and 5.1c).
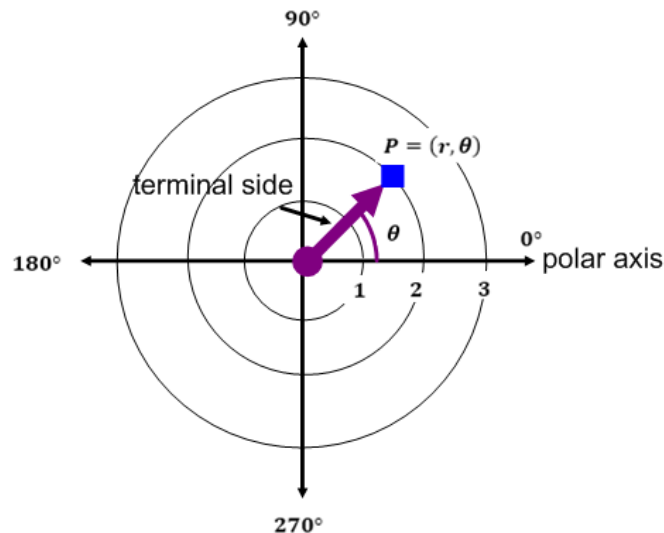

Figure 5.1b: Polar coordinate (r, θ) measures in degrees.

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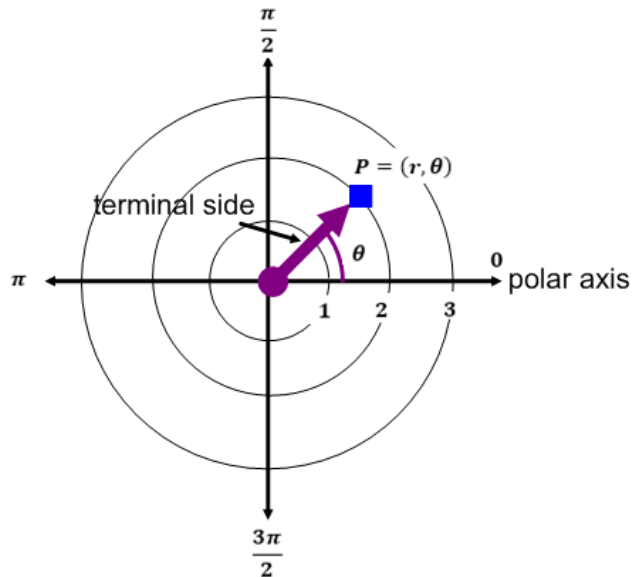CENTER FOR
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Figure 5.1c: Polar coordinate (r, θ) measures in radians.

The angle θ in the polar coordinate can be measured in degrees or in radians. If θ is positive, the terminal side rotates counterclockwise from $0°$. If θ is negative, the terminal side rotates clockwise from $0°$. The point P is located r units from the pole. If r is positive, the point P lies on the terminal side of θ. If r is negative, P lies on the ray that is opposite to the terminal side. If r is equal to zero, P lies on the pole regardless the value of θ.

EXAMPLES

Please work through the following examples before completing the 5.1 LEARNING ACTIVITY:

Example 1: Plot $(3, 120°)$.

$120°$ is located in the second quadrant. We will rotate the terminal side counterclockwise from the polar axis. Since r is positive, P lies on the terminal side 3 units from the pole.
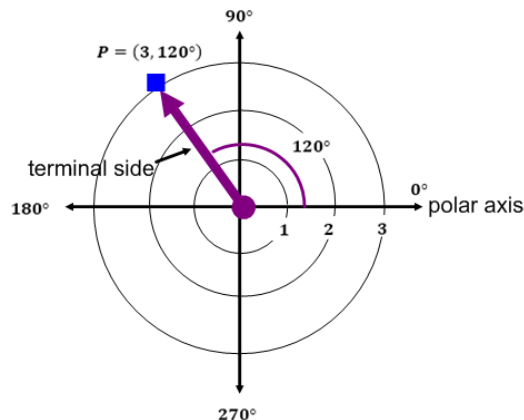
Example 2: Plot $\left(-2, \frac{\pi}{4}\right)$.

$\frac{\pi}{4}$ is located in the first quadrant. We will rotate the terminal side counterclockwise from the polar axis. Since r is negative, P lies on the ray that is opposite to the terminal side.
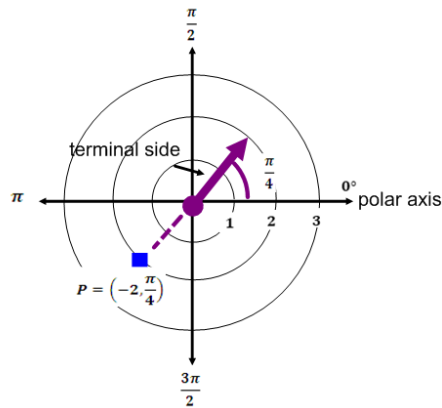
VIDEO 5.1A	VIDEO 5.1A TRANSCRIPT
Click here to watch an example of how to plot polar coordinate.	Transcript of video available under Module 5 in Moodle.

5.1 LEARNING ACTIVITY

a. Plot $\left(2, \frac{\pi}{4}\right)$.

b. Plot $\left(-3, \frac{4\pi}{3}\right)$.

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5.2 CONVERTING BETWEEN RECTANGULAR AND POLAR COORDINATES

If we examine the rectangular and polar coordinate system together, we see that the *x*-axis coincides with the polar axis and the origin coincides with the pole (See figure 5.2a).
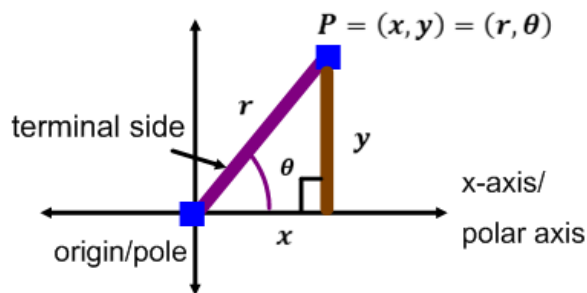

Figure 5.2a: Polar coordinates coincides with rectangular coordinates.

The point P has rectangular coordinates (x, y) and polar coordinates (r, θ). So, from figure 5.2a, we can conclude the hypotenuse $r^2 = x^2 + y^2$, $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$, and $\tan\theta = \frac{y}{r}$. Thus, the relationship between the rectangular coordinate system and the polar coordinate system can be described in table 5.2a.

Table 5.2a

Relationship Between Rectangular and Polar Coordinate

$$r^2 = x^2 + y^2$$

$$\cos\theta = \frac{x}{r}, \qquad x = r \cdot \cos\theta$$

$$\sin\theta = \frac{y}{r}, \qquad y = r \cdot \sin\theta$$

$$\tan\theta = \frac{y}{x}$$

To convert between rectangular and polar coordinates, we will use the equations described in table 5.2a.

EXAMPLES

Please work through the following examples before completing the 5.2 LEARNING ACTIVITY:

Example 1: Convert $\left(3, \frac{3\pi}{2}\right)$ to a rectangular coordinate.

$\left(3, \frac{3\pi}{2}\right)$ **is in the form of (r, θ). We will use $x = r \cdot \cos\theta$ and $y = r \cdot \sin\theta$.**

$$x = r \cdot \cos\theta$$

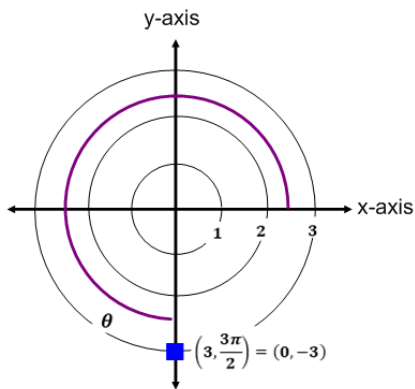$$x = 3 \cdot \cos\frac{3\pi}{2} \qquad \cos\frac{3\pi}{2} = 0$$

$$x = 0$$

$$y = r \cdot \sin\theta$$

$$y = 3 \cdot \sin\frac{3\pi}{2} \qquad \sin\frac{3\pi}{2} = -1$$

$$y = -3$$

So, the polar coordinate $\left(3, \frac{3\pi}{2}\right)$ is the rectangular coordinate $(0, -3)$.

Example 2: Convert $\left(2, \frac{\pi}{3}\right)$ to a rectangular coordinate.

$\left(2, \frac{\pi}{3}\right)$ is in the form of (r, θ). We will use $x = r \cdot \cos\theta$ and $y = r \cdot \sin\theta$.

$$x = r \cdot \cos\theta$$

$$x = 2 \cdot \cos\frac{\pi}{3} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

$$x = 1$$

$$y = r \cdot \sin\theta$$

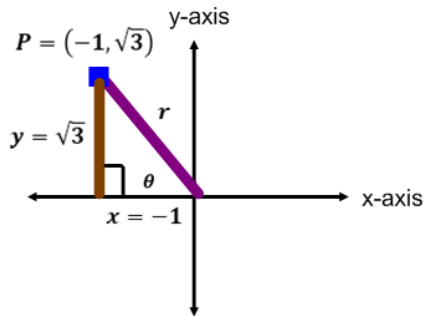$$y = 2 \cdot \sin\frac{\pi}{3} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$y = \sqrt{3}$$

So, the polar coordinate $\left(2, \frac{\pi}{3}\right)$ is the rectangular coordinate $(2, \sqrt{3})$.

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Example 3: Convert $\left(-1, \sqrt{3}\right)$ to a polar coordinate.

To convert $\left(-1, \sqrt{3}\right)$ to polar coordinate, we will begin by plotting $\left(-1, \sqrt{3}\right)$ in the second quadrant, use the Pythagorean Theorem to find r, and use tangent to find θ.

$r^2 = x^2 + y^2$ **take a square root on both sides**

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2}$

$r = \sqrt{1 + 3} = \sqrt{4}$

$r = 2$

$\tan \theta = \dfrac{y}{x} = \dfrac{\sqrt{3}}{-1} = -\sqrt{3}$

Since θ lies in the second quadrant and $\tan\dfrac{\pi}{3} = \sqrt{3}$, by the reference angle, $\theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$.

So, the rectangular coordinate $\left(-1, \sqrt{3}\right)$ is the polar coordinate $\left(2, \dfrac{2\pi}{3}\right)$.

5.2 LEARNING ACTIVITY

a. Convert $(3, \pi)$ to a rectangular coordinate.

b. Convert $\left(1, -\sqrt{3}\right)$ to polar coordinate.

5.3 CONVERTING A RECTANGULAR EQUATION TO A POLAR EQUATION

A *polar equation* is an equation that contains variables r and θ. To convert rectangular equations into polar equations, we will replace x with $r \cdot \cos \theta$ and y with $r \cdot \sin \theta$.

EXAMPLES

Please work through the following examples before completing the 5.3 LEARNING ACTIVITY:

Example 1: Convert $3x - y = 6$ to a polar equation.

$$3x - y = 6 \quad \text{replace x with } r \cdot \cos\theta \text{ and y with } r \cdot \sin\theta$$

$$3 \cdot r \cos\theta - r \cdot \sin\theta = 6 \quad \text{factor r}$$

$$r(3\cos\theta - \sin\theta) = 6 \quad \text{divide both sides by } 3\cos\theta - \sin\theta$$

$$r = \frac{6}{3\cos\theta - \sin\theta}$$

So, the rectangular equation $3x - y = 6$ is the polar equation $r = \dfrac{6}{3\cos\theta - \sin\theta}$.

Example 2: Convert $(x - 1)^2 + y^2 = 1$ to a polar equation.

$$(x - 1)^2 + y^2 = 1 \quad \text{replace x with } r \cdot \cos\theta \text{ and y with } r \cdot \sin\theta$$

$$(r \cdot \cos\theta - 1)^2 + (r \cdot \sin\theta)^2 = 1 \quad \text{multiply } (r \cdot \cos\theta - 1)^2 \text{ and } (r \cdot \sin\theta)^2$$

$$r^2\cos^2\theta - 2r\cos\theta + 1 + r^2\sin^2\theta = 1 \quad \text{subtract 1 on both sides}$$

$$r^2\cos^2\theta + r^2\sin^2\theta - 2r\cos\theta = 0 \quad \text{factor } r^2$$

$$r^2(\cos^2\theta + \sin^2\theta) - 2r\cos\theta = 0 \quad \text{use Pythagrean Identity } \cos^2\theta + \sin^2\theta = 1$$

$$r^2 \cdot 1 - 2r\cos\theta = 0 \quad \text{factor r}$$

$$r \cdot (r - 2\cos\theta) = 0 \quad \text{set each factor equal to zero}$$

$$r = 0, \ r - 2\cos\theta = 0 \quad \text{solve for r}$$

$$r = 2\cos\theta$$

Although $r = 0$ is a single point at the pole, it is not necessary to include $r = 0$ as part of the answer. So, the rectangular equation $(x - 1)^2 + y^2 = 1$ is the polar equation $r = 2\cos\theta$.

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5.3 LEARNING ACTIVITY

a. Convert $2x + y = 3$ to a polar equation.

b. Convert $(x - 2)^2 + y^2 = 4$ to a polar equation.

5.4 CONVERTING A POLAR EQUATION TO A RECTANGULAR EQUATION

When converting equations from polar to rectangular, we want to obtain an equation in which the variables are x and y rather than r and θ. We can use the equations such as $r^2 = x^2 + y^2$, $\cos\theta = \frac{x}{r}$, $x = r \cdot \cos\theta$, $\sin\theta = \frac{y}{r}$, $y = r \cdot \sin\theta$, and $\tan\theta = \frac{y}{x}$ from table 5.2a to convert a polar equation to a rectangular equation.

EXAMPLES

Please work through the following examples before completing the 5.4 LEARNING ACTIVITY:

Example 1: Convert $r = 4$ to a rectangular equation.

    We can use $r^2 = x^2 + y^2$ to convert $r = 4$ to a rectangular equation.

        $r = 4$ **square both sides**

        $r^2 = 16$ **replace r^2 with $x^2 + y^2$**

        $x^2 + y^2 = 16$

    So, the polar equation $r = 4$ is the rectangular equation $x^2 + y^2 = 16$.

Example 2: Convert $\theta = \frac{\pi}{3}$ to a rectangular equation.

    We can use $\tan\theta = \frac{y}{x}$ to convert $\theta = \frac{\pi}{3}$ to a rectangular equation.

        $\theta = \dfrac{\pi}{3}$ **take a tangent on both sides**

        $\tan\theta = \tan\dfrac{\pi}{3}$ **recall $\tan\dfrac{\pi}{3} = \sqrt{3}$**

        $\tan\theta = \sqrt{3}$ **replace $\tan\theta$ with $\dfrac{y}{x}$**

        $\dfrac{y}{x} = \sqrt{3}$ **multiply x on both sides**

        $y = \sqrt{3}x$

    So, the polar equation $\tan\theta = \frac{y}{x}$ is the rectangular equation $y = \sqrt{3}x$.

Example 3: Convert $r = 4\csc\theta$ to a rectangular equation.

We can use $y = r \cdot \sin\theta$ to convert $r = 4\csc\theta$ to a rectangular equation.

$r = 4\csc\theta$ write $\csc\theta$ as $\sin\theta$

$r = \dfrac{4}{\sin\theta}$ multiply both sides by $\sin\theta$

$r \cdot \sin\theta = 4$ replace $r\sin\theta$ with y

$y = 4$

So, the polar equation $r = 4\csc\theta$ is the rectangular equation $y = 4$.

Example 4: Convert $r = 6\cos\theta$ to a rectangular equation.

We can use $r^2 = x^2 + y^2$ to convert $r = 6\cos\theta$ to a rectangular equation.

$r = 6\cos\theta$ multiply both sides by r

$r^2 = 6r\cos\theta$ replace r^2 with $x^2 + y^2$ and $r\cos\theta$ with x

$x^2 + y^2 = 6x$ subtract 6x on both sides

$x^2 - 6x + y^2 = 0$ complete the squares for $x^2 - 6x$

$x^2 - 6x + 9 + y^2 = 0 + 9$ factor $x^2 - 6x + 9$

$(x - 3)^2 + y^2 = 9$

So, the polar equation $r = 6\cos\theta$ is the rectangular equation $(x - 3)^2 + y^2 = 9$.

5.4 LEARNING ACTIVITY

a. Convert $r = 5$ to a rectangular equation.

b. Convert $\theta = \dfrac{\pi}{6}$ to a rectangular equation.

c. Convert $r = 3\sec\theta$ to a rectangular equation.

d. Convert $r = 4\sin\theta$ to a rectangular equation.

5.5 GRAPHS OF POLAR EQUATIONS

Polar equations can be graphed on a graphing calculator. The graphs of polar equations consist of circle graphs, rose curves, limacons, and lemniscates.

The circle graphs in polar coordinates have the form of $r = a \cos \theta$ and $r = a \sin \theta$ (see figure 5.5a and 5.5b).
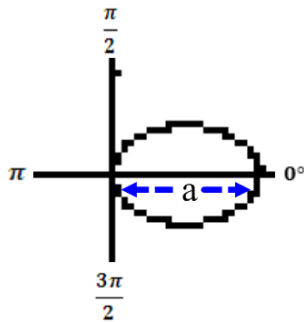
Figure 5.5a: A circle graph in the form of $r = a \cos \theta$ where θ is in radians.
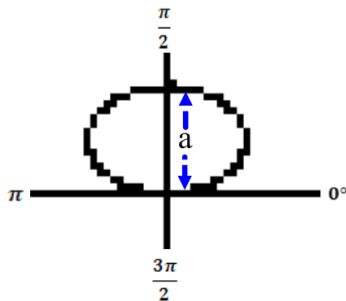
Figure 5.5b: A circle graph in the form of $r = a \sin \theta$ where θ is in radians.

The graph of limacons has the form of $r = a + b \sin \theta$, $r = a - b \sin \theta$, $r = a + b \cos \theta$, and $r = a - b \cos \theta$ where $a > 0$, $b > 0$, and $\frac{a}{b}$ determines a limacon's shape (see 5.5c through 5.5f). If $\frac{a}{b} < 1$, then the limacon has an inner loop. If $\frac{a}{b} = 1$, then the limacon is a heart shaped. If $1 < \frac{a}{b} < 2$, then the limacon is dimpled with no inner loop. If $\frac{a}{b} = 2$, then the limacon has no dimple and no inner loop.
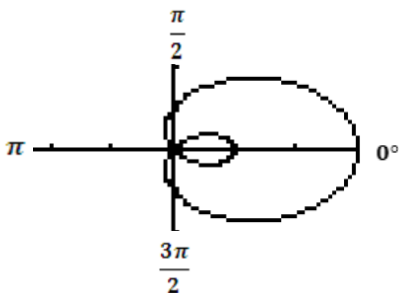
Figure 5.5c: The graph of a limacon $r = 1 + 2 \cos \theta$ with an inner loop.

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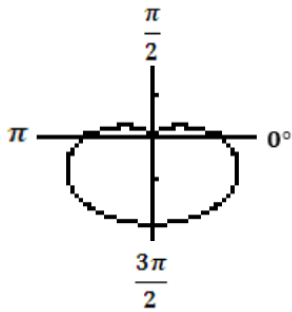
Figure 5.5d: The graph of a limacon $r = 1 - \sin\theta$ that is heart shaped.
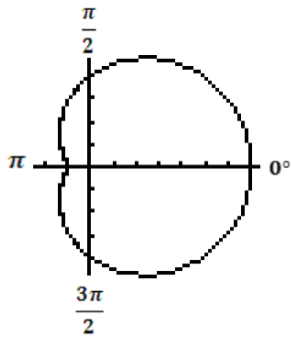
Figure 5.5e: The graph of a limacon $r = 4 + 3\cos\theta$ that is dimpled with no inner loop.
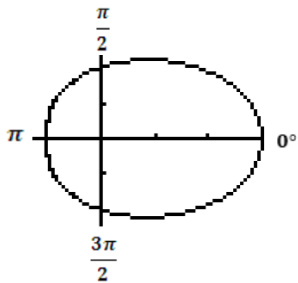
Figure 5.5f: The graph of a limacon $r = 2 + \cos\theta$ with no dimpled and no inner loop.

The graph of rose curves has the form $r = a\sin n\theta$ and $r = a\cos n\theta$ (see figure 5.5g and 5.5h). If n is odd, then the rose curve has n petals. If n is even, then the rose curve has $2n$ petals.
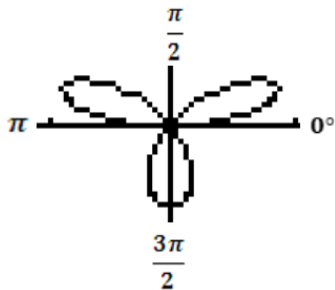

Figure 5.5g: The graph of a rose curve $r = \sin 3\theta$ with 3 petals where n is odd.

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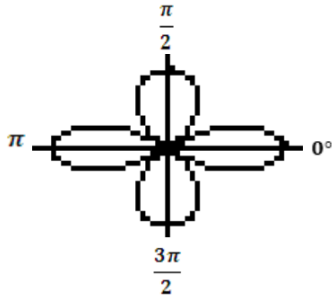
Figure 5.5h: The graph of a rose curve $r = \cos 2\theta$ with 4 petals where n is even.

The graph of lemniscates has the form $r^2 = a^2 \sin 2\theta$ and $r^2 = a^2 \cos 2\theta$ (see figure 5.5i and 5.5j). The graph of $r^2 = a^2 \sin 2\theta$ is symmetric to the pole where as the graph of $r^2 = a^2 \cos 2\theta$ is symmetric to the polar axis, $\theta = \frac{\pi}{2}$, and the pole.
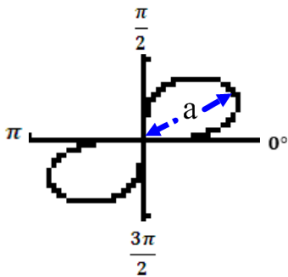
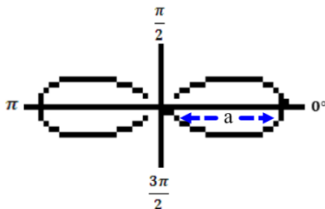Figure 5.5i: A lemniscate graph with the form $r^2 = a^2 \sin 2\theta$ that is symmetric to the pole.

Figure 5.5j: A lemniscate graph with the form $r^2 = a^2 \cos 2\theta$ that is symmetric to the polar axis, $\theta = \frac{\pi}{2}$, and the pole.

To use a graphing calculator to graph equations in the polar coordinate, we will first set the calculator in polar mode and then set the angle in radian mode. Let's follow the steps below to set up the calculator.

Step 1: Press

MODE

Step 2: Select

POL

Step 3: Select

RADIAN

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Once we set up the calculator, to graph a polar equation, let's follow the steps described below.

Step 1: Press

Y=
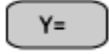
Step 2: Enter the polar equation.

Step 3: Press

WINDOW

Step 4: Set the appropriate viewing window. We will make $\theta step$ relative small so the calculator will plot more points. If the Xmin, Xmax, Ymin, and Ymax are too small or too large, we can adjust these numbers.

$\theta\text{min} = 0$

$\theta\text{max} = 2\pi$

$\theta\text{step} = \dfrac{\pi}{96}$

$X\text{min} = -5$

$X\text{max} = 5$

$X\text{scl} = 1$

$Y\text{min} = -5$

$Y\text{max} = 5$

$Y\text{scl} = 1$

Step 5: Press

GRAPH

EXAMPLES

Please work through the following examples before completing the 5.5 LEARNING ACTIVITY:

Example 1: Graph $r = 3\cos\theta$ using a graphing calculator.

> ***Step 1: Press***

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Y=

Step 2: Enter the polar equation.

3 COS X,T,θ,n)

Step 3: Press

WINDOW

Step 4: Set the appropriate viewing window.

θmin $= 0$

θmax $= 2\pi$

θstep $= \dfrac{\pi}{96}$

Xmin $= -3$

Xmax $= 3$

Xscl $= 1$

Ymin $= -3$

Ymax $= 3$

Yscl $= 1$
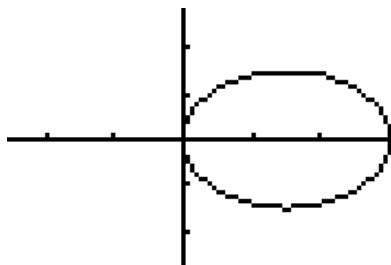
Step 5: Press

GRAPH

Example 2: Graph $r = 2 - \sin\theta$ **using a graphing calculator.**

Step 1: **Press**

Y=

Step 2: **Enter the polar equation.**

2 — SIN X,T,Θ,n)

Step 3: **Press**

WINDOW

Step 4: **Set the appropriate viewing window.**

$\theta \text{min} = 0$

$\theta \text{max} = 2\pi$

$\theta \text{step} = \frac{\pi}{96}$

$X\text{min} = -3$

$X\text{max} = 3$

$X\text{scl} = 1$

$Y\text{min} = -3$

$Y\text{max} = 3$

$Y\text{scl} = 1$

Step 5: **Press**

GRAPH

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Example 3: **Graph** $r = 2\cos 2\theta$ **using a graphing calculator.**
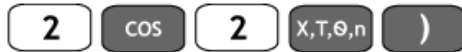
Step 1: **Press**

Y=

Step 2: **Enter the polar equation.**

2 | COS | 2 | X,T,Θ,n |)

Step 3: **Press**

WINDOW

Step 4: **Set the appropriate viewing window.**

$\theta\text{min} = 0$

$\theta\text{max} = 2\pi$

$\theta\text{step} = \dfrac{\pi}{96}$

$X\text{min} = -3$

$X\text{max} = 3$

$X\text{scl} = 1$

$Y\text{min} = -3$

$Y\text{max} = 3$

$Y\text{scl} = 1$

Step 5: **Press**

GRAPH

Example 4: Graph $r^2 = 9 \sin 2\theta$ using a graphing calculator.

Step 1: **Press**

Y=

Step 2: **Enter the polar equation.**

2nd x^2 9   SIN   2   X,T,Θ,n   )

Step 3: **Press**

WINDOW

Step 4: **Set the appropriate viewing window.**

$\theta\text{min} = 0$

$\theta\text{max} = 2\pi$

$\theta\text{step} = \dfrac{\pi}{96}$

$X\text{min} = -3$

$X\text{max} = 3$

$X\text{scl} = 1$

$Y\text{min} = -3$

$Y\text{max} = 3$

$Y\text{scl} = 1$

Step 5: **Press**

GRAPH

5.5 LEARNING ACTIVITY

a. Graph $r = 2 \sin \theta$ using a graphing calculator.

b. Graph $r = 1 + \sin \theta$ using a graphing calculator.

c. Graph $r = 3 \sin 2\theta$ using a graphing calculator.

d. Graph $r^2 = 16 \sin 2\theta$ using a graphing calculator.

MAJOR CONCEPTS

KEY CONCEPTS

A polar coordinate system is just like a Unit Circle on the rectangular coordinate system where the *x*-axis becomes a polar axis, and the origin becomes a pole.

To convert between rectangular and polar coordinates, we use $r^2 = x^2 + y^2$, $\cos \theta = \frac{x}{r}$, $\sin \theta = \frac{y}{r}$, and $\tan \theta = \frac{y}{r}$.

To convert rectangular equations into polar equations, we will replace x with $r \cdot \cos \theta$ and y with $r \cdot \sin \theta$.

We can use the equations $r^2 = x^2 + y^2$, $\cos \theta = \frac{x}{r}$, $x = r \cdot \cos \theta$, $\sin \theta = \frac{y}{r}$, $y = r \cdot \sin \theta$, and $\tan \theta = \frac{y}{x}$ to convert a polar equation to a rectangular equation.

The circle graphs in polar coordinates have the form of $r = a \cos \theta$ and $r = a \sin \theta$.

The graph of limacons has the form of $r = a + b \sin \theta$, $r = a - b \sin \theta$, $r = a + b \cos \theta$, and $r = a - b \cos \theta$ where $a > 0$, $b > 0$, and $\frac{a}{b}$ determines a limacon's shape.

The graph of rose curves has the form $r = a \sin n\theta$ and $r = a \cos n\theta$. If n is odd, then the rose curve has n petals. If n is even, then the rose curve has $2n$ petals.

The graph of lemniscates has the form $r^2 = a^2 \sin 2\theta$ and $r^2 = a^2 \cos 2\theta$.

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KEY TERMS

Polar Coordinate	Polar Axis	Pole	Polar Equation

GLOSSARY

A *polar coordinate* system is just like a Unit Circle on the rectangular coordinate system where the *x*-axis becomes a polar axis, and the origin becomes a pole.

A *polar axis* is a ray that extends to the right of the polar coordinate system.

The *pole* is the endpoint of the polar axis.

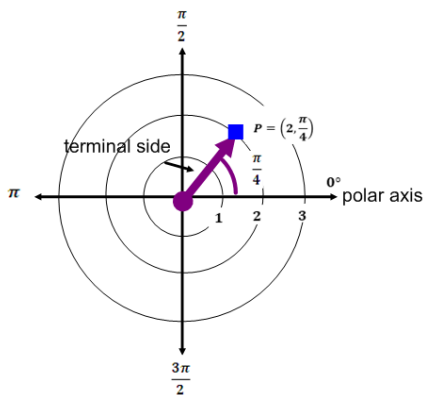A *polar equation* is an equation that contains variables r and θ.

ASSESSMENT

ANSWERS TO LEARNING ACTIVITIES

5.1 LEARNING ACTIVITY

a. Plot $\left(2, \frac{\pi}{4}\right)$.

   Ans:

b. Plot $\left(-3, \frac{4\pi}{3}\right)$.

   Ans:
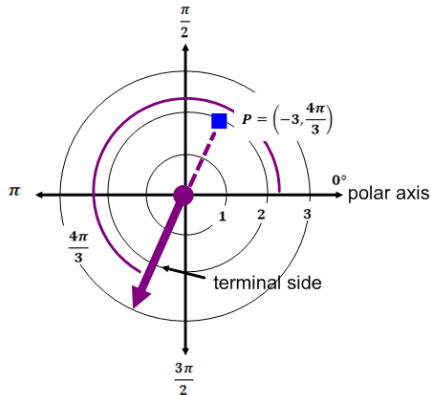
5.2 LEARNING ACTIVITY

a. Convert $(3, \pi)$ to a rectangular coordinate.
    Ans: $(-3, 0)$

b. Convert $(1, -\sqrt{3})$ to a polar coordinate.
    Ans: $\left(2, \dfrac{5\pi}{3}\right)$

5.3 LEARNING ACTIVITY

a. Convert $2x + y = 3$ to a polar equation.
    Ans: $r = \dfrac{3}{2\cos\theta + \sin\theta}$

b. Convert $(x - 2)^2 + y^2 = 4$ to a polar equation.
    Ans: $r = 4\cos\theta$

5.4 LEARNING ACTIVITY

a. Convert $r = 5$ to a rectangular equation.
    Ans: $x^2 + y^2 = 25$

b. Convert $\theta = \dfrac{\pi}{6}$ to a rectangular equation.
    Ans: $y = \dfrac{\sqrt{3}}{3}x$

c. Convert $r = 3\sec\theta$ to a rectangular equation.
    Ans: $x = 3$

d. Convert $r = 4 \sin \theta$ to a rectangular equation.
   Ans: $x^2 + (y - 2)^2 = 4$

5.5 LEARNING ACTIVITY

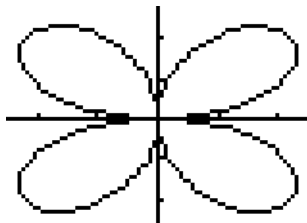a. Graph $r = 2 \sin \theta$ using a graphing calculator.
   Ans:
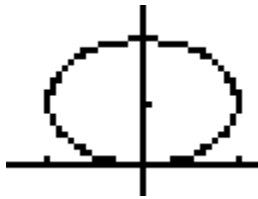
b. Graph $r = 1 + \sin \theta$ using a graphing calculator.
   Ans:
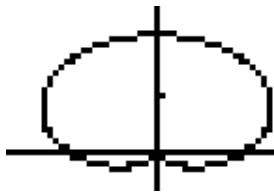
c. Graph $r = 3 \sin 2\theta$ using a graphing calculator.
   Ans:

d. Graph $r^2 = 16 \sin 2\theta$ using a graphing calculator.
   Ans:

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MODULE REINFORCEMENT

SHORT ANSWER QUESTIONS

1) Plot $\left(-2, \frac{\pi}{2}\right)$.

2) Plot $(1, \pi)$.

3) Convert $\left(2, \frac{\pi}{6}\right)$ to a rectangular coordinate.

4) Convert $(-2, 2)$ to a polar coordinate.

5) Convert $3x + y = 4$ to a polar equation.

6) Convert $(x-3)^2 + y^2 = 9$ to a polar equation.

7) Convert $r = 2$ to a rectangular equation.

8) Convert $\theta = \frac{2\pi}{3}$ to a rectangular equation.

9) Convert $r = 4\sec\theta$ to a rectangular equation.

10) Convert $r = 2\sin\theta$ to a rectangular equation.

MULTIPLE CHOICE: READ THE FOLLOWING QUESTIONS OR STATEMENTS AND SELECT THE BEST ANSWER.

1) _____ Select the correct graph of $\left(3, \frac{\pi}{6}\right)$.
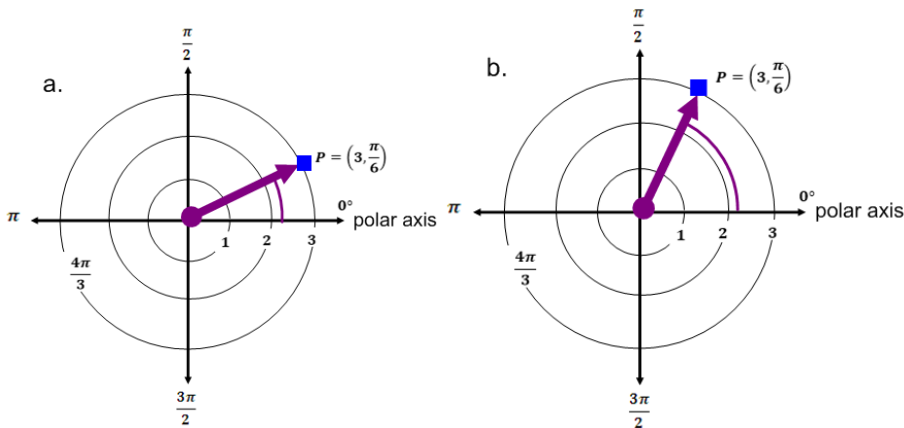
2) _____ Select the correct graph of $\left(-2, \frac{\pi}{3}\right)$.
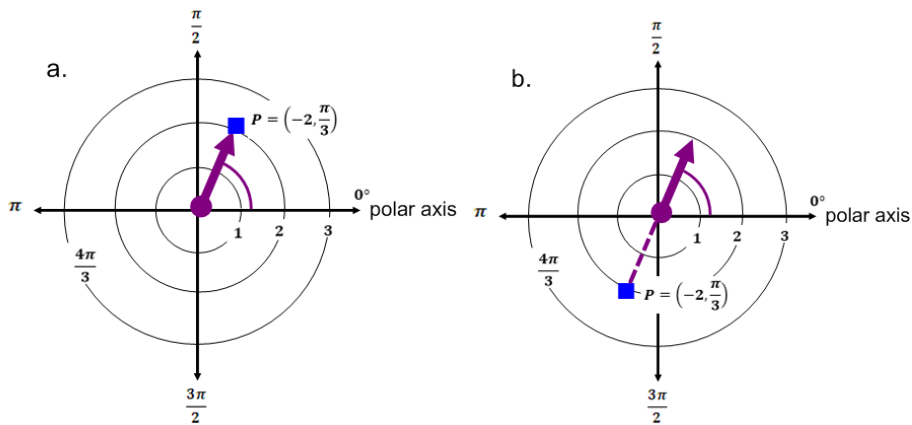
3) _____ Convert $\left(1, \frac{\pi}{4}\right)$ to a rectangular coordinate.

   a. $\left(\frac{\sqrt{2}}{2}, 1\right)$

   b. $\left(1, \frac{\sqrt{2}}{2}\right)$

   c. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

4) _____ Convert $\left(2, -2\sqrt{3}\right)$ to a polar coordinate.

    a. $\left(4, \frac{\pi}{3}\right)$

    b. $\left(4, \frac{2\pi}{3}\right)$

    c. $\left(4, \frac{4\pi}{3}\right)$

    d. $\left(4, \frac{5\pi}{3}\right)$

5) _____ Convert $x + y = 2$ to a polar equation.

    a. $\frac{2}{\cos\theta + \sin\theta}$

    b. $\frac{2}{\cos\theta - \sin\theta}$

    c. $r\cos\theta + r\sin\theta$

6) _____ Convert $x^2 + (y - 4)^2 = 16$ to a polar equation.

    a. $r - 4\sin\theta = 0$

    b. $r - 8\sin\theta = 0$

    c. $r + 4\sin\theta = 0$

7) _____ Convert $r = -1$ to a rectangular equation.

    a. $x^2 + y^2 = -1$

    b. $x^2 - y^2 = -1$

    c. $x^2 + y^2 = 1$

8) _____ Convert $\theta = \frac{\pi}{4}$ to a rectangular equation.

    a. $y = \frac{\sqrt{2}}{2}x$

    b. $y = x$

9) _____ Convert $r = 2\csc\theta$ to a rectangular equation.
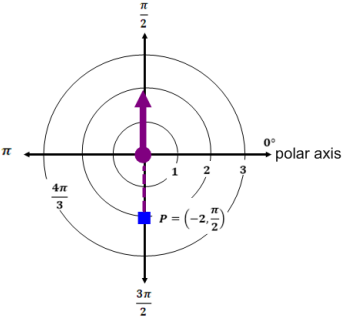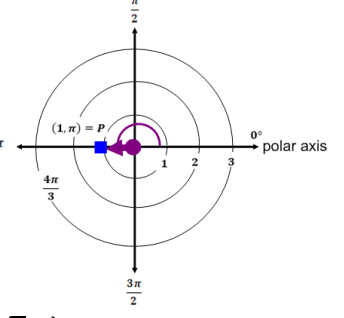
    a. $x = 2$

    b. $y = 2$

10) _____ Convert $r = 2\cos\theta$ to a rectangular equation.

    a. $(x - 2)^2 + y^2 = 4$

    b. $x^2 + (y - 1)^2 = 1$

    c. $(x - 1)^2 + y^2 = 1$

ANSWER KEY

| Short Answer Questions	Multiple Choice
1) *Ans*: 2) *Ans*: 3) $\left(\sqrt{3}, 1\right)$ 4) $\left(2\sqrt{2}, \frac{3\pi}{4}\right)$ 5) $\dfrac{4}{3\cos\theta + \sin\theta}$ 6) $r = 6\cos\theta$ 7) $x^2 + y^2 = 4$ 8) $y = -\sqrt{3}x$ 9) $x = 4$ 10) $x^2 + (y-1)^2 = 1$	1) a 2) b 3) c 4) d 5) a 6) b 7) c 8) b 9) b 10) c

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CRITICAL THINKING

1) When plotting polar coordinates, if r is negative, why does point P lie on the ray opposite to the terminal side?

   Ans: r is the positve distance from the pole as θ rotates counterclockwise or clockwise.

   When r is negativ, we have to plot on the other side of the pole.
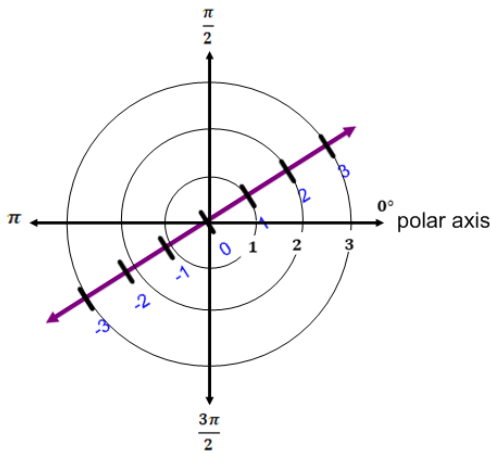
2) Why is $\left(2, \frac{\pi}{3}\right)$ the same representation as $\left(2, \frac{7\pi}{3}\right)$?

   Ans: If the terminal side makes one rotation counterclockwise from $\frac{\pi}{3}$, which is adding 2π to $\frac{\pi}{3}$,

   it will rotate back to $\left(2, \frac{\pi}{3}\right)$