

# 6

## Conic Sections

### Objectives

Students will be able to:

- Define the twelve key terms regarding conic sections.
- Write the equation of a circle, ellipse, hyperbola, and parabola in the standard form.
- Identify the center, vertex, and focus from the equation of an ellipse, hyperbola, and parabola.

### Orienting Questions

- ✓ What are the definitions of the twelve key terms in this module?
- ✓ How are the equations of a circle, ellipse, hyperbola, and parabola written in the standard form?
- ✓ How are the center, vertex, and focus of an ellipse, hyperbola, and parabola identified?



## INTRODUCTION

Nature is a great place to find examples of curves. If we take a look at a beautiful rainbow after a summer thunderstorm, we can see it has the shape of a parabola or if we study the movement of the planets, we have to study curves. As a result of these naturally occurring curves, mathematicians throughout history have studied the different types of curves. The study of curves is also involved in manmade objects such as tunnels, bridges, and navigation systems. To study curves you have to use the concept of conic sections. **Conic sections** are curves that are created by the intersection of a right circular cone and a plane. The conic sections we will examine in this module are circle, ellipse, hyperbola, and parabola.

## 6.1 CIRCLE

A **circle** is a set of points in a plane where the points have equal distance to the center. A radius is the distance from the center to any point on a circle (See Figure 6.1a). The standard form of a circle is  $(x - h)^2 + (y - k)^2 = r^2$  where  $(x, y)$  represents any point on the circle and  $(h, k)$  represents the center.

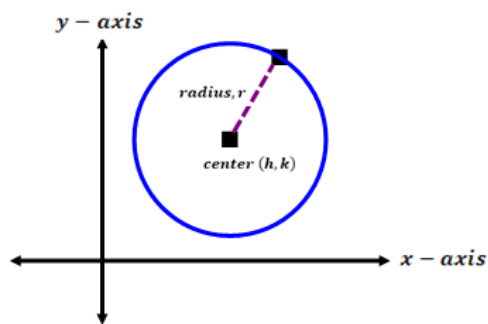


Figure 6.1a: A circle in the form of  $(x - h)^2 + (y - k)^2 = r^2$  consists of a center and a radius.

## EXAMPLES

*Please work through the following examples before completing the 6.1 LEARNING ACTIVITY:*

**Example 1: Find the equation of a circle centered at  $(-2, 2)$  with a radius 3.**

Since the center  $(-2, 2)$  represents  $(h, k)$ ,  $h = -2$  and  $k = 2$ . With a radius  $r = 3$ , we can plug into the standard form.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-2))^2 + (y - 2)^2 = (3)^2$$

$$(x + 2)^2 + (y - 2)^2 = 9$$



**Example 2:** For the equation of a circle  $(x + 3)^2 + (y - 1)^2 = 16$ , find the center and the radius.

Since the equation  $(x - (-3))^2 + (y - 1)^2 = (4)^2$  is in the form of  $(x - h)^2 + (y - k)^2 = r^2$ ,  $h = -3$ ,  $k = 1$ , and  $r = 4$ .

#### VIDEO 6.1A

Click [here](#) to watch an example of how to write the equation of a circle.

#### VIDEO 6.1A TRANSCRIPT

Transcript is available under Module 6 in Moodle.

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### 6.1 LEARNING ACTIVITY

- Find the equation of a circle centered at  $(3,2)$  with a radius 2.
- For the equation of a circle  $(x + 3)^2 + (y + 3)^2 = 9$ , find the center and the radius.



## 6.2 ELLIPSE

An **ellipse** is the set of points in a plane where the sum of the distance from two fixed points are constant (See figure 6.2a).

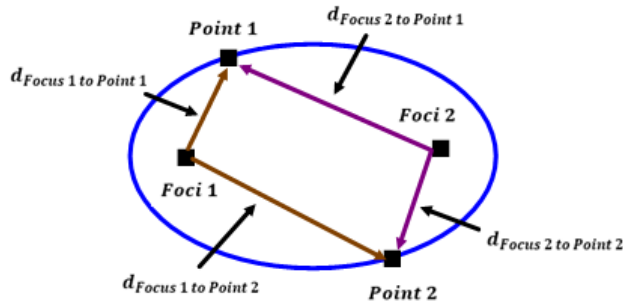


Figure 6.2a: An ellipse where  
 $d_{\text{Focus 1 to Point 1}} + d_{\text{Focus 1 to Point 2}}$   
 $= d_{\text{Focus 2 to Point 1}} + d_{\text{Focus 2 to Point 2}}$

**Foci** (singular: **Focus**) are two fixed points inside the ellipse. The **center** of an ellipse is the midpoint connecting the focus. The **major axis** is a line segment that passes through the foci and joins the vertices. The **minor axis** is a line segment, whose endpoints are on the ellipse, which is perpendicular to the major axis at the center (See figure 6.2b). The **vertices** are the endpoints of the major axis.

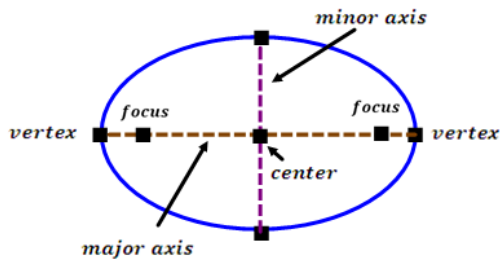


Figure 6.2b: An ellipse consists of a center, vertices, foci, a major axis, and a minor axis.

An ellipse has the standard form  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  and  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ . The ellipse is centered at  $(h, k)$ . In the standard form  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  where  $a^2 > b^2$ , the foci are  $c$  units to the right and left of the center and  $c^2 = a^2 - b^2$ . The major axis is parallel to the  $x$ -axis. The endpoints of the major axis are  $a$  units to the right and left of the center and the minor axis are  $b$  units above and below the center (See figure 6.2c).



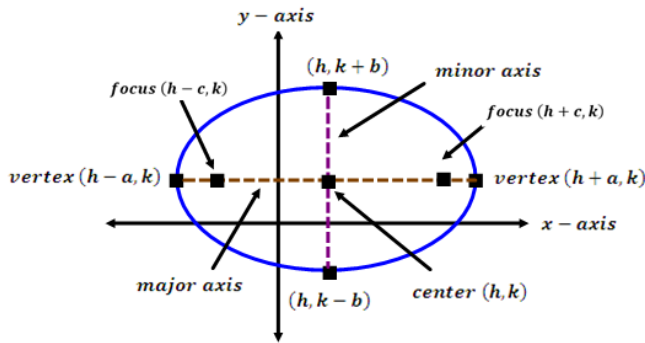


Figure 6.2c: An ellipse in the form of  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  where  $a^2 > b^2$ .

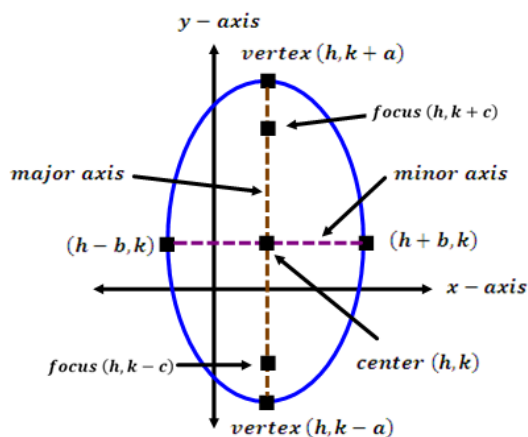


Figure 6.2d: An ellipse in the form of  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$  where  $a^2 > b^2$ .

In the standard form  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$  where  $a^2 > b^2$ , the foci are  $c$  units above and below the center and  $c^2 = a^2 - b^2$ . The major axis is parallel to the  $y$ -axis. The endpoints of the major axis are  $a$  units above and below the center and the minor axis are  $b$  units to the right and left of the center (See figure 6.2d).

## EXAMPLES

*Please work through the following examples before completing the 6.2 LEARNING ACTIVITY:*

**Example 1:** For the equation of an ellipse  $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{9} = 1$ , find the center, vertices, the endpoints of the minor axis, and foci.

$$\frac{(x-1)^2}{(2)^2} + \frac{(y-(-3))^2}{(3)^2} = 1 \text{ is in the form of } \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \text{ where } a = 3, b = 2, h = 1, \text{ and } k = -3.$$

Since  $a^2 > b^2$ , the major axis is parallel to the  $y$ -axis.  $(h, k)$  is the center,  $(h, k + a)$  and  $(h, k - a)$  are the vertices,  $(h + b, k)$  and  $(h - b, k)$  are the endpoints of the minor axis, and  $(h, k + c)$   $(h, k - c)$  are the foci.

$$(h, k) = (1, -3) \text{ is the center.}$$

$(h, k + a) = (1, -3 + 3) = (1, 0)$  and  $(h, k - a) = (1, -3 - 3) = (1, -6)$  are the vertices.

$(h + b, k) = (1 + 2, -3) = (3, -3)$  and  $(h - b, k) = (1 - 2, -3) = (-1, -3)$  are the endpoints of the minor axis.

Since the foci are  $c$  units above and below the center, we use  $c^2 = a^2 - b^2$  to find  $c$ .

$$c^2 = a^2 - b^2 \quad \text{take a square root on both sides}$$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{(3)^2 - (2)^2}$$

$$c = \sqrt{9 - 4} = \sqrt{5}$$

$(h, k + c) = (1, -3 + \sqrt{5})$  and  $(1, -3 - \sqrt{5})$  are the foci.

**Example 2:** Find the equation of an ellipse where the foci are  $(5, 0)$ ,  $(-5, 0)$  and the vertices are  $(7, 0)$ ,  $(-7, 0)$ .

To find the equation of an ellipse in the form of  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ , we will find the center  $(h, k)$ ,  $a^2$  and  $b^2$ .

The vertices  $(7, 0)$ ,  $(-7, 0)$  are  $a$  units from the center,  $a = 7$ . Thus,  $a^2 = 49$ .

The vertex  $(h + a, k) = (7, 0)$ , where  $k = 0$ , is 7 units to the right of the center. We will substitute 7 for  $a$  and set  $h + a$  equal to 7 to solve for  $h$ .

$$h + a = 7 \quad \text{substitute 7 for a}$$

$$h + 7 = 7 \quad \text{subtract 7 on both sides}$$

$$h = 0$$

Since the foci  $(5, 0)$ ,  $(-5, 0)$  are  $c$  units from the center,  $c = 5$ . So, to find  $b^2$ , we use  $c^2 = a^2 - b^2$ .

$$c^2 = a^2 - b^2 \quad \text{substitute 5 for c and 7 for a}$$

$$5^2 = 7^2 - b^2$$

$$25 = 49 - b^2 \quad \text{add } b^2 \text{ and subtract 25 on both sides}$$

$$b^2 = 49 - 25$$

$$b^2 = 24$$



So, the equation is  $\frac{x^2}{49} + \frac{y^2}{24} = 1$ .

**Example 3:** Find the equation of an ellipse that has a horizontal major axis with length 10, a minor axis with length 4, and the center located at  $(-2, 3)$ .

To find the equation of an ellipse in the form of  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ , we will find the center  $(h, k)$ ,  $a^2$  and  $b^2$ .

The center  $(h, k) = (-2, 3)$ , so  $h = -2$  and  $k = 3$ .

Since the center is the midpoint for the major and minor axis,

$$a = 5 \text{ and } b = 2$$

$$a^2 = 25 \text{ and } b^2 = 4$$

Thus, the equation is  $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{4} = 1$ .

## 6.2 LEARNING ACTIVITY

- For the equation of an ellipse  $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1$ , find the center, vertices, the endpoints of the minor axis, and foci.
- Find the equation of an ellipse where the foci are  $(-2, 0)$ ,  $(2, 0)$  and the vertices are  $(-4, 0)$ ,  $(4, 0)$ .
- Find the equation of an ellipse that has a vertical major axis with length 8, a minor axis with length 4, and the center located at  $(1, 3)$ .



### 6.3 HYPERBOLA

A **hyperbola** is the set of points in a plane where the differences of the distance from two foci are constant (See figure 6.3a).

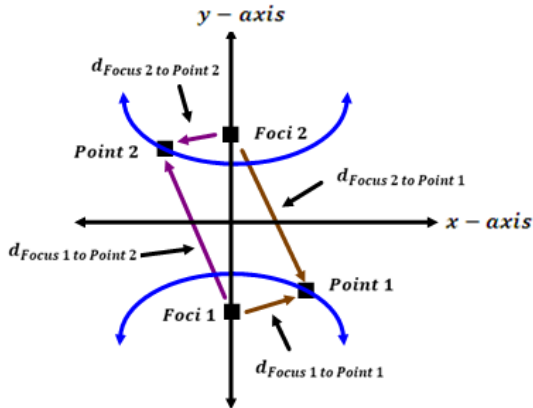


Figure 6.3a: A hyperbola where  
 $d_{Focus 2 to Point 1} - d_{Focus 1 to Point 1}$   
 $= d_{Focus 1 to Point 2} - d_{Focus 2 to Point 2}$

The **transverse axis** is a line segment that joins the vertices. The midpoint of the transverse axis is the center  $(h, k)$  of the hyperbola. The asymptotes of a hyperbola are two straight lines that pass through the center separating the two branches (See figure 6.3b and 6.3c).

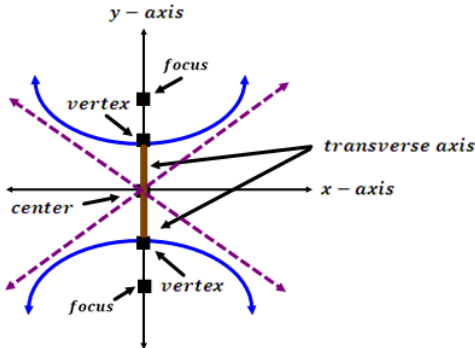


Figure 6.3b: The midpoint of the transverse axis is the center of the hyperbola.

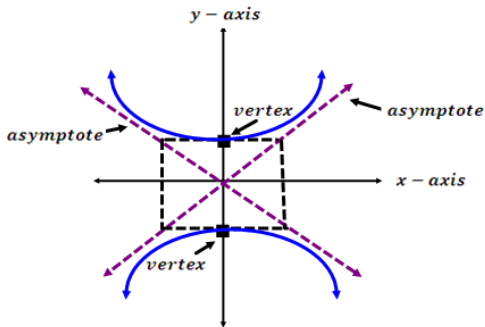


Figure 6.3c: The two asymptotes pass through the center of a hyperbola.



A hyperbola has the standard form  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$  and  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ . In the standard form  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ , the foci are  $c$  units to the right and left of the center where  $c^2 = a^2 + b^2$ . The transverse axis is parallel to the  $x$ -axis. The vertices are  $a$  units to the right and left of the center. If the hyperbola is centered at the origin, the two asymptotes have equations that are  $y = \frac{b}{a}x$  and  $y = -\frac{b}{a}x$  (See figure 6.3d). In the standard form  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ , the foci are  $c$  units above and below the center where  $c^2 = a^2 + b^2$ . The transverse axis is parallel to the  $y$ -axis. The vertices are  $a$  units above and below the center. If the hyperbola is centered at the origin, the two asymptotes have equations that are  $y = \frac{a}{b}x$  and  $y = -\frac{a}{b}x$  (See figure 6.3e).

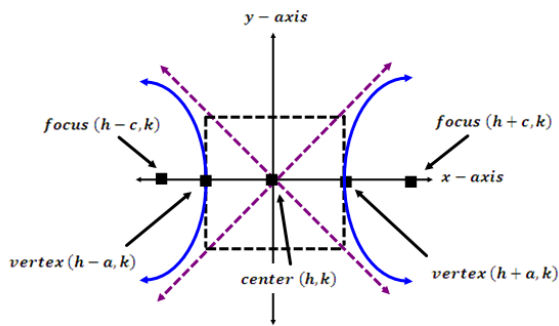


Figure 6.3d: A hyperbola in the form of  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ .

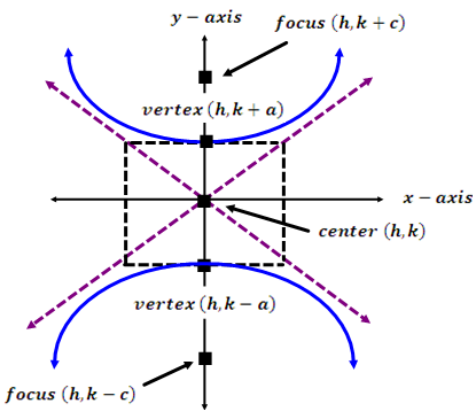


Figure 6.3e: A hyperbola in the form of  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ .

## EXAMPLES

*Please work through the following examples before completing the 6.3 LEARNING ACTIVITY:*

**Example 1:** For the equation of a hyperbola  $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{4} = 1$ , find the center, vertices, foci, and the two asymptotes.

Since  $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{4} = 1$  is in the form of  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ ,  $h = 2$ ,  $k = 3$ ,  $a = 4$ , and  $b = 2$ .



$(h, k)$  is the center,  $(h + a, k)$  and  $(h - a, k)$  are the vertices,  $(h + c, k)$  and  $(h - c, k)$  are the foci.

$(h, k) = (2, 3)$  is the center.

$(h + a, k) = (2 + 4, 3) = (6, 3)$  and  $(h - a, k) = (2 - 4, 3) = (-2, 3)$  are the vertices.

To find the foci  $(h + c, k)$ , we use  $c^2 = a^2 + b^2$  to find  $c$ .

$c^2 = a^2 + b^2$  take a square root on both sides

$c = \sqrt{a^2 + b^2}$  substitute 4 for a and 2 for b

$$c = \sqrt{(4)^2 + (2)^2}$$

$$c = \sqrt{16 + 4} = \sqrt{20} \text{ simplify } \sqrt{20}$$

$$c = 2\sqrt{5}$$

$(h + c, k) = (2 + 2\sqrt{5}, 3)$  and  $(h - c, k) = (2 - 2\sqrt{5}, 3)$  are the foci.

Since the center of the hyperbola is shifted to  $(2, 3)$ , the two asymptotes are

$$y - 3 = \frac{4}{2}(x - 2) \text{ and } y - 3 = -\frac{4}{2}(x - 2).$$

$$y - 3 = \frac{4}{2}(x - 2) \quad y - 3 = -\frac{4}{2}(x - 2)$$

$$y - 3 = 2(x - 2) \quad y - 3 = -2(x - 2) \text{ distribute}$$

$$y - 3 = 2x - 4 \quad y - 3 = -2x + 4$$

$$y = 2x - 4 + 3 \quad y = -2x + 4 + 3 \text{ add 3 on both sides}$$

$$y = 2x - 1 \quad y = -2x + 7$$

**Example 2:** Find the equation of a hyperbola where  $(4, -1)$  is the center,  $(7, -1)$  is the focus, and  $(6, -1)$  is the vertex.

To find the equation of a hyperbola in the form of  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$  or  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ , we will need to find  $h$ ,  $k$ ,  $a^2$ , and  $b^2$ .

The center  $(h, k) = (4, -1)$ . So,  $h = 4$  and  $k = -1$ .

Since the vertex and the focus have the same  $y$ -value as the center, we use

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ form to find the equation.}$$



The vertex  $(h + a, k) = (6, -1)$  is 6 units to the right of the vertex. We will substitute 4 for  $h$ , set  $h + a$  equal to 6 to solve for  $a$ , and find  $a^2$ .

$$h + a = 6 \quad \text{substitute 4 for h}$$

$$4 + a = 6 \quad \text{subtract 4 on both sides}$$

$$a = 2$$

$$a^2 = 4$$

The focus  $(h + c, k) = (7, -1)$  is 7 units above the center. We will substitute 4 for  $h$ , set  $h + a$  equal to 7 to solve for  $c$ , and find  $c^2$ . Once we solved for  $c^2$ , we will use  $c^2 = a^2 + b^2$  to find  $b^2$ .

$$h + c = 7 \quad \text{substitute 4 for h}$$

$$4 + c = 7 \quad \text{subtract 4 on both sides}$$

$$c = 3$$

$$c^2 = 9$$

$$c^2 = a^2 + b^2 \quad \text{subtract } a^2 \text{ on both sides}$$

$$c^2 - a^2 = b^2 \quad \text{substitute 4 for a and 9 for c}$$

$$9 - 4 = b^2$$

$$5 = b^2$$

So, the equation is  $\frac{(x-4)^2}{4} - \frac{(y+1)^2}{5} = 1$ .

### 6.3 LEARNING ACTIVITY

- For the equation of a hyperbola  $\frac{(x-3)^2}{16} - \frac{(y-3)^2}{4} = 1$ , find the center, vertices, foci, and the two asymptotes.
- Find the equation of a hyperbola where  $(-2,1)$  is the center,  $(-2,5)$  is the focus, and  $(-2,3)$  is the vertex.



## 6.4 PARABOLA

A **parabola** is the set of points in a plane that have equal distance from a fixed line called directrix and the focus (See figure 6.4a). The **latus rectum** is a line segment that passes through the focus and parallel to the directrix. The endpoints of the latus rectum are on the parabola, and its length is equal to  $|4p|$  (See figure 6.4b).

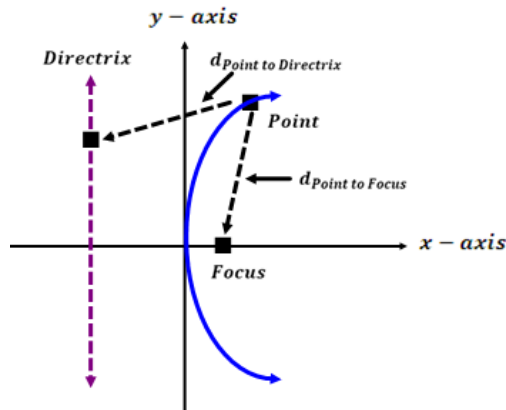


Figure 6.4a: A parabola where  $d_{\text{Point to Focus}} = d_{\text{Point to Directrix}}$

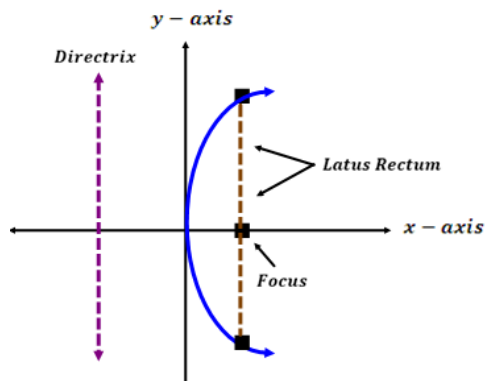


Figure 6.4b: The latus rectum of a parabola that passes through the focus.

The standard form of a parabola is  $(y - k)^2 = 4p(x - h)$  and  $(x - h)^2 = 4p(y - k)$ . In the standard form  $(y - k)^2 = 4p(x - h)$ , the vertex is  $(h, k)$ , the focus is  $(h + p, k)$ , and the directrix is  $x = h - p$  (See figure 6.4c). If  $p > 0$ , the parabola opens to the right. If  $p < 0$ , the parabola opens to the left. In the standard form  $(x - h)^2 = 4p(y - k)$ , the vertex is  $(h, k)$ , the focus is  $(h, k + p)$ , and the directrix is  $y = k - p$  (See figure 6.4d). If  $p > 0$ , the parabola opens upward. If  $p < 0$ , the parabola opens downward.



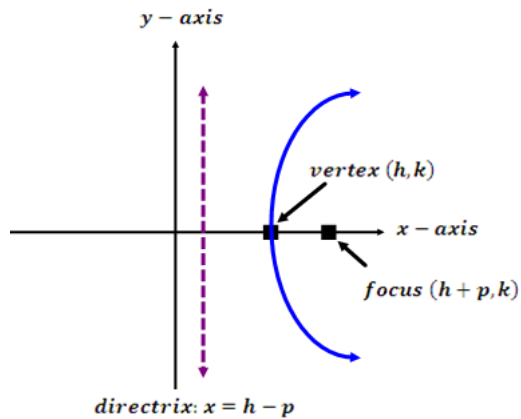


Figure 6.3c: A parabola in the form of  $(y - k)^2 = 4p(x - h)$  with  $p > 0$ .

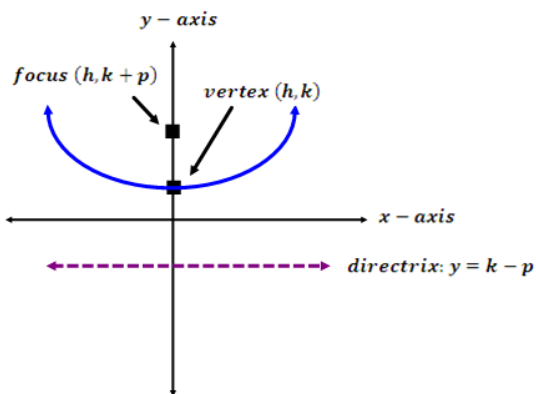


Figure 6.3d: A parabola in the form of  $(x - h)^2 = 4p(y - k)$  with  $p > 0$ .

## EXAMPLES

Please work through the following examples before completing the 6.4 LEARNING ACTIVITY:

**Example 1:** For the equation of a parabola  $(x - 2)^2 = 4(y - 1)$ , find the vertex, focus, and directrix.

Since  $(x - 2)^2 = 4(y - 1)$  is in the form of  $(x - h)^2 = 4p(y - k)$ , the vertex is  $(h, k)$ , the focus is  $(h, k + p)$ , the directrix is  $y = k - p$ ,  $h = 2$ , and  $k = 1$ . To find  $p$ , we set  $4p$  equal to 4 and solve for  $p$ .

$$4p = 4 \quad \text{divide both sides by 4}$$

$$p = 1$$

$(h, k) = (2, 1)$  is the vertex.

$(h, k + p) = (2, 1 + 1) = (2, 2)$  is the focus.

$y = k - p = 1 - 1 = 0$  is the directrix.

**Example 2:** Find the equation of a parabola where the focus is  $(3, 2)$  and the directrix is  $x = -5$ .

To find the equation of a parabola in the form of  $(y - k)^2 = 4p(x - h)$  or  $(x - h)^2 = 4p(y - k)$ , we will need to find  $h$ ,  $k$ , and  $p$ . Since the directrix  $x = h - p$  is a vertical line  $x = -5$ , we use  $(y - k)^2 = 4p(x - h)$  form.

Since the focus  $(h + p, k) = (3, 2)$ ,  $k = 2$  and  $h + p = 3$ . Because  $h$  and  $p$  are unknown, we can use the equation from the focus  $h + p = 3$  and the equation from directrix  $-5 = h - p$  to help us to find  $h$  and  $p$ .

$$\begin{aligned} -5 &= h - p \\ h + p &= 3 \end{aligned} \quad \text{solve for } p \text{ from the second equation by subtracting } h \text{ on both sides}$$

$$\begin{aligned} -5 &= h - p \\ p &= 3 - h \end{aligned} \quad \text{substitute } p \text{ in the first equation for } 3 - h$$

$$-5 = h - (3 - h) \quad \text{combine like terms}$$

$$-5 = 2h - 3 \quad \text{add 3 on both sides}$$

$$-5 + 3 = 2h$$

$$-2 = 2h \quad \text{divide both sides by 2}$$

$$h = -1$$

Substitute  $h = -1$  into  $h + p = 3$  to find  $p$ .

$$-1 + p = 3 \quad \text{add 1 both sides}$$

$$p = 4$$

So, the equation is  $(y - 2)^2 = 16(x + 1)$ .

#### 6.4 LEARNING ACTIVITY

- For the equation of a parabola  $(x - 2)^2 = 4(y - 1)$ , find the vertex, focus, and directrix.
- Find the equation of a parabola where the focus is  $(2, 3)$  and the directrix is  $x = -2$ .



## MAJOR CONCEPTS

## KEY CONCEPTS

The study of conic sections includes the circle, ellipse, hyperbola, and parabola.

To write the equation of a circle in the standard form  $(x - h)^2 + (y - k)^2 = r^2$ , we will need to find the center  $(h, k)$  and the radius  $r$ .

In the standard form of an ellipse  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  where  $a^2 > b^2$ , the center is  $(h, k)$ , the vertices are  $(h - a, k)$ ,  $(h + a, k)$ , the foci are  $(h - c, k)$ ,  $(h + c, k)$ , the endpoints of minor axis are  $(h, k - b)$ ,  $(h, k + b)$ , and  $c^2 = a^2 - b^2$ .

In the standard form of an ellipse  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$  where  $a^2 > b^2$ , the center is  $(h, k)$ , the vertices are  $(h, k - a)$ ,  $(h, k + a)$ , the foci are  $(h, k - c)$ ,  $(h, k + c)$ , the endpoints of minor axis are  $(h - b, k)$ ,  $(h + b, k)$ , and  $c^2 = a^2 - b^2$ .

In the standard form of a hyperbola  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ , the center is  $(h, k)$ , the vertices are  $(h - a, k)$ ,  $(h + a, k)$ , the foci are  $(h - c, k)$ ,  $(h + c, k)$ ,  $c^2 = a^2 + b^2$ , and the asymptotes are  $y = \frac{b}{a}x$  and  $y = -\frac{b}{a}x$ .

In the standard form of a hyperbola  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ , the center is  $(h, k)$ , the vertices are  $(h, k - a)$ ,  $(h, k + a)$ , the foci are  $(h, k - c)$ ,  $(h, k + c)$ ,  $c^2 = a^2 + b^2$ , and the asymptotes are  $y = \frac{a}{b}x$  and  $y = -\frac{a}{b}x$ .

In the standard form of a parabola  $(y - k)^2 = 4p(x - h)$  where  $p > 0$ , the vertex is  $(h, k)$ , the focus is  $(h + p, k)$ , and the directrix is  $x = h - p$ .

In the standard form of a parabola  $(x - h)^2 = 4p(y - k)$  where  $p > 0$ , the vertex is  $(h, k)$ , the focus is  $(h, k + p)$ , and the directrix is  $y = k - p$ .

## KEY TERMS

Conic Sections	Circle	Ellipse	Focus (Foci)
Center	Major axis	Minor axis	Vertices (Vertex)
Hyperbola	Transverse Axis	Parabola	Latus Rectum



## GLOSSARY

**Conic sections** are curves that are created by the intersection of a right circular cone and a plane.

A **circle** is a set of points in a plane where the points have equal distance to the center.

An **ellipse** is the set of points in a plane where the sum of the distance from two fixed points are constant.

**Foci** (singular: **Focus**) are two fixed points inside of an ellipse.

The **center** of an ellipse is the midpoint connecting the focus.

The **major axis** of an ellipse is a line segment that passes through the foci and joins the vertices.

The **minor axis** of an ellipse is a line segment, whose endpoints are on the ellipse, which is perpendicular to the major axis at the center.

The **vertices** of an ellipse are the endpoints of the major axis.

A **hyperbola** is the set of points in a plane where the differences of the distance from two foci are constant.

The **transverse axis** of a hyperbola is a line segment that joins the vertices.

A **parabola** is the set of points in a plane that have equal distance from a fixed line called directrix and the focus.

The **latus rectum** of a parabola is a line segment that passes through the focus and parallel to the directrix.





## ASSESSMENT

## ANSWERS TO LEARNING ACTIVITIES

## 6.1 LEARNING ACTIVITY

- a. Find the equation of a circle centered at (3,2) with a radius 2.

**Ans:**  $(x - 3)^2 + (y - 2)^2 = 4$

- b. For the equation of a circle  $(x + 3)^2 + (y + 3)^2 = 9$ , find the center and the radius.

**Ans:** *Center*  $(-3, -3)$ ,  $r = 3$

## 6.2 LEARNING ACTIVITY

- a. For the equation of an ellipse  $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1$ , find the center, vertices, the endpoints of the minor axis, and foci.

**Ans:** *center is*  $(2, 1)$ , *vertices are*  $(-1, 1)$ ,  $(5, 1)$ , *endpoints of minor axis are*  $(2, 3)$ ,  $(2, -1)$ , *foci*  $(2 - \sqrt{5}, 1)$ ,  $(2 + \sqrt{5}, 1)$

- b. Find the equation of an ellipse where the foci are  $(-2,0)$ ,  $(2,0)$  and the vertices are  $(-4,0)$ ,  $(4,0)$ .

**Ans:**  $\frac{x^2}{4} + \frac{y^2}{12} = 1$

- c. Find the equation of an ellipse that has a vertical major axis with length 8, a minor axis with length 4, and the center located at  $(1,3)$ .

**Ans:**  $\frac{(x - 1)^2}{4} + \frac{(y - 3)^2}{16} = 1$

## 6.3 LEARNING ACTIVITY

- a. For the equation of a hyperbola  $\frac{(x-3)^2}{16} - \frac{(y-3)^2}{4} = 1$ , find the center, vertices, foci, and the two asymptotes.

**Ans:** *center is*  $(3, 3)$ , *vertices are*  $(-1, 3)$ ,  $(7, 3)$ , *foci are*  $(3 - 2\sqrt{5}, 3)$ ,  $(3 + \sqrt{5}, 3)$ ,  
*asymptotes are*  $y = \frac{1}{2}x + \frac{3}{2}$ ,  $y = -\frac{1}{2}x + \frac{9}{2}$

- b. Find the equation of a hyperbola where  $(-2,1)$  is the center,  $(-2,5)$  is the focus, and  $(-2,3)$  is the vertex.

**Ans:**  $\frac{(y - 1)^2}{4} - \frac{(x + 2)^2}{12} = 1$



**6.4 LEARNING ACTIVITY**

- a. For the equation of a parabola  $(x - 2)^2 = 4(y - 1)$ , find the vertex, focus, and directrix.

*Ans: vertex is (2, 1), focus is (2, 0), directrix  $y = 0$*

- b. Find the equation of a parabola where the focus is (2,3) and the directrix is  $x = -2$ .

*Ans:  $(y - 3)^2 = 8(x - 0)$*



**MODULE REINFORCEMENT****SHORT ANSWER QUESTIONS**

- 1) Find the equation of a circle centered at  $(-1,0)$  with a radius 3.
- 2) For the equation of a circle  $(x - 1)^2 + (y - 2)^2 = 16$ , find the center and the radius.
- 3) For the equation of the an ellipse  $\frac{(x+3)^2}{25} + \frac{(y-3)^2}{16} = 1$ , find the vertices.
- 4) Find the equation of an ellipse where the foci are  $(0, -2), (0,2)$  and the vertices are  $(0, -4), (0,4)$ .
- 5) Find the equation of an ellipse that has a horizontal major axis with length 6, a minor axis with length 4, and the center located at  $(3,4)$ .
- 6) For the equation of a hyperbola  $\frac{y^2}{16} - \frac{(x+2)^2}{36} = 1$ , find the foci.
- 7) Find the equation of a hyperbola where  $(3, -4)$  is the center,  $(3,4)$  is the focus, and  $(3,2)$  is the vertex.
- 8) For the equation of a parabola  $(y - 2)^2 = 4(x - 1)$ , find the focus.
- 9) For the equation of a parabola  $(y - 2)^2 = 4(x - 1)$ , find the directrix.
- 10) Find the equation of a parabola where the focus is  $(2,3)$  and the directrix is  $y = -1$ .



**MULTIPLE CHOICE: READ THE FOLLOWING QUESTIONS OR STATEMENTS AND SELECT THE BEST ANSWER.**

- 1) \_\_\_\_\_ Choose the equation of a circle centered at  $(-1,4)$  with a radius 4.
- $(x - 1)^2 + (y - 4)^2 = 16$
  - $(x + 1)^2 + (y - 4)^2 = 4$
  - $(x + 1)^2 + (y - 4)^2 = 16$
- 2) \_\_\_\_\_ For the equation of a circle  $(x + 1)^2 + y^2 = 36$ , find the center and the radius.
- Center  $(-1,0)$ ,  $r = 6$
  - Center  $(0, -1)$ ,  $r = 6$
  - Center  $(-1,0)$ ,  $r = 36$
- 3) \_\_\_\_\_ For the equation of an ellipse  $\frac{(x-1)^2}{4} + \frac{(y+1)^2}{16} = 1$ , find the vertices.
- $(5, -1)$ ,  $(-3, -1)$
  - $(1,3)$ ,  $(1, -5)$
  - $(5, -1)$ ,  $(1, -5)$
- 4) \_\_\_\_\_ Find the equation of an ellipse where the foci are  $(-5,0)$ ,  $(5,0)$  and the vertices are  $(-6,0)$ ,  $(6,0)$ .
- $\frac{x^2}{6} + \frac{y^2}{11} = 1$
  - $\frac{x^2}{11} + \frac{y^2}{36} = 1$
  - $\frac{x^2}{36} + \frac{y^2}{11} = 1$
- 5) \_\_\_\_\_ Find the equation of an ellipse that has a vertical major axis with length 12, a minor axis with length 6, and the center located at  $(-1, -2)$ .
- $\frac{(x-1)^2}{36} + \frac{(y-2)^2}{9} = 1$
  - $\frac{(x+1)^2}{9} + \frac{(y+2)^2}{36} = 1$
  - $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{36} = 1$



- 6) \_\_\_\_\_ For the equation of a hyperbola  $\frac{x^2}{49} - \frac{(y-1)^2}{16} = 1$ , find the foci.
- $(1, -\sqrt{65}), (1, \sqrt{65})$
  - $(-7, 1), (7, 1)$
  - $(-\sqrt{65}, 1), (\sqrt{65}, 1)$
- 7) \_\_\_\_\_ Find the equation of a hyperbola where  $(-3, -3)$  is the center,  $(-3, 7)$  is the focus, and  $(-3, -5)$  is the vertex.
- $\frac{(y-3)^2}{4} - \frac{(x-3)^2}{96} = 1$
  - $\frac{(y+3)^2}{96} - \frac{(x+3)^2}{4} = 1$
  - $\frac{(y+3)^2}{4} - \frac{(x+3)^2}{96} = 1$
- 8) For the equation of a parabola  $(y + 2)^2 = 4(x + 1)$ , find the focus.
- $(0, -2)$
  - $(-1, -2)$
  - $(1, 2)$
- 9) For the equation of a parabola  $(y + 2)^2 = 4(x + 1)$ , find the directrix.
- $x = 2$
  - $x = -2$
  - $x = 1$
- 10) Find the equation of a parabola where the focus is  $(-3, 4)$  and the directrix is  $y = 2$ .
- $(x + 3)^2 = 4(y - 4)$
  - $(y + 3)^2 = 4(x - 3)$
  - $(x + 3)^2 = 4(y - 3)$

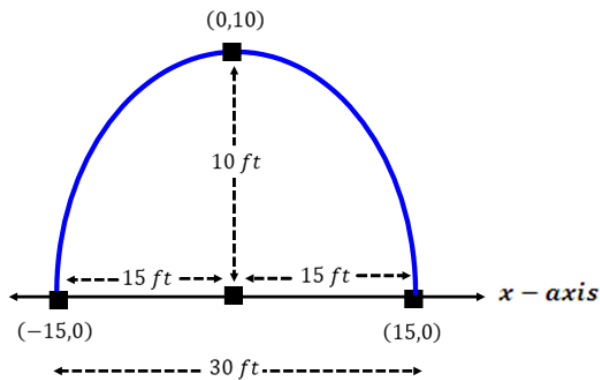


## ANSWER KEY

Short Answer Questions	Multiple Choice
1) $(x + 1)^2 + y^2 = 9$	1) c
2) Center (1,2), $r = 4$	2) a
3) (2,3), (-8,3)	3) b
4) $\frac{x^2}{12} + \frac{y^2}{16} = 1$	4) c
5) $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{4} = 1$	5) b
6) $(-2, -2\sqrt{13}), (-2, 2\sqrt{13})$	6) c
7) $\frac{(y+4)^2}{36} - \frac{(x-3)^2}{28} = 1$	7) c
8) (2,2)	8) a
9) $x = 0$	9) b
10) $(x - 2)^2 = 8(y - 1)$	10) c

## CRITICAL THINKING

- 1) Suppose a truck 8 ft wide and 8 ft tall is entering a semielliptical arched tunnel that is 10 ft tall and 30 ft wide. Can the truck drive through the tunnel without going into the other lane (See figure below)?



*Ans:*

*Since the truck is 8 ft wide, it corresponds to  $x = 8$ . We can find the height of the arch way 8 ft from the center by substituting 8 for  $x$ . We will then determine if the height of the truck exceeds the height of the tunnel 8 ft away from the center.*

$$\frac{x^2}{15^2} + \frac{y^2}{10^2} = 1 \quad \text{this is equation of the ellipse}$$

$$\frac{8^2}{15^2} + \frac{y^2}{10^2} = 1 \quad \text{substitute 8 for } x$$

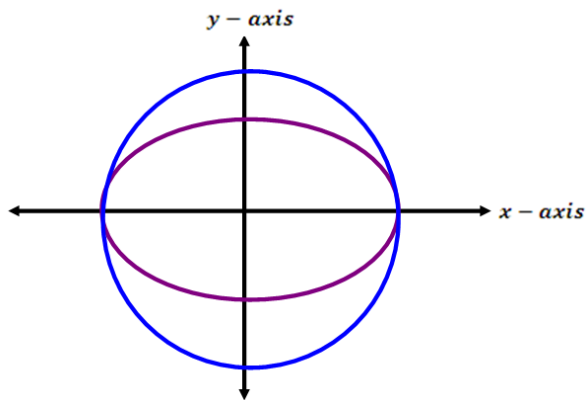


$$\frac{64}{225} + \frac{y^2}{100} = 1 \quad \text{solve for } y$$

$$y \approx 8.4 \text{ ft}$$

*The height 8 ft away from the center is less than 10 ft. The truck can drive through the tunnel without going into the other lane.*

- 2) The equation of the purple ellipse is  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ . Find the equation of the blue circle.



*Ans:*

$$x^2 + y^2 = 16$$

- 3) Why is the length of the latus rectum  $|4p|$ ?

*Answers may vary.*

*If the vertex  $(h, k)$  is located at the origin, then the focus  $(h + p, k) = (p, 0)$  and the equation of the parabola is  $y^2 = 4px$ .*

*Since the endpoints of latus rectum is above and below the focus, the  $x$ -value of the endpoints are  $p$ . So, we can substitute  $p$  for  $x$  into the equation and find the  $y$ -values of the endpoints.*

$$y^2 = 4px \quad \text{substitute } p \text{ for } x$$

$$y^2 = 4pp$$

$$y^2 = 4p^2 \quad \text{take a square root on both sides}$$

$$y = \pm 2p$$

*The vertical distance from one endpoint to the focus is  $2p$ . Thus, the total distance is  $|4p|$ .*



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